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THE EFFECT OF TERMINAL CONSTANTS ON TELEPHONE TRANSMISSION

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THE EFFECT OF TERMINAL CONSTANTS ON TELEPHONE TRANSMISSION

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A Thesis Submitted for
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CHAPTER I.

Introduction.

Since the advent of the telephone into the commercial world telephone engineers have been constantly turning their efforts toward securing greater efficiency of speech transmission. At first the main diffioulties arose in the telephone instruments themselves, due to the crude types of transmitters and receivers used at that time. The sensitiveness of the early transmitters was greatly improved by the Blake transmitter and finally a still more sensitive transmitter was placed on the market, viz;- the solid back type. The receivers also underwent changes until finally the bipolar type has been looked upon as the best adapted for practical purposes. Further improvements in the construction of the transmitter and receiver have been at a standstill for several years. It has been found that a highly sensitive receiver is of no particular advantage because both desirable and undesirable sounds are magnified equally.

The early telephone lines were constructed having a ground return, but this was found to be very disadvantageous, due to the many disturbances in the circuit. Mr. Carty, now of the Amercian Telephone and Telegraph Company, suggested the use of an entire metallic circuit. With the use of the metallic circuit the speech transmission was greatly improved over that of the ground return due to the fact that the line disturbances were greatly reduced. Iron and steel wire were used at first but finally the hard drawn copper wire was brought into use and was found to be a great improvement over the iron and steel wire,

owing to the fact that the line losses were greatly reduced.

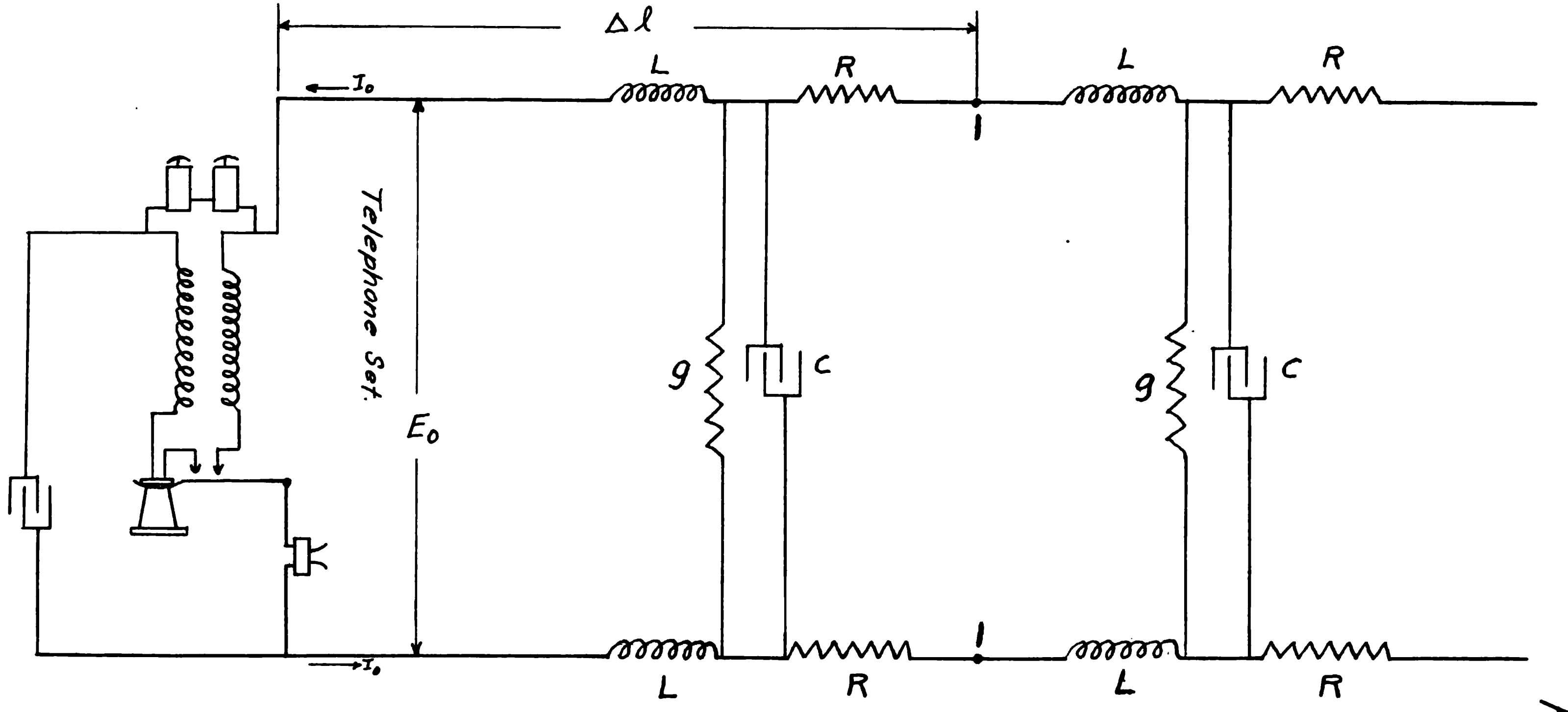
During the early years of the telephone there was little necessity for the study of long distance telephony because all communication over the telephone was carried on within the limits of one city and the distance of transmission was limited to a few miles. However, it was not long before the demand arose for inter-communication between cities. Up to this time the main difficulties encountered had been with imperfect transmitting and receiving apparatus and with disturbances in the line due to induction from power wires, stray currents, etc. But with the introduction of the long-distance line, new, unheard of difficulties made themselves apparent. It was found that even on the best lines speech could not be transmitted more than one-hundred miles. This fact lead such men as Sir Oliver Heaviside and Professor M. I. Pupin into a study of the long distance telephone line, and through their efforts, especially those of the latter the distance of speech transmission has been lengthened many fold.

Professor Pupin devised an entirely new means of improving speech transmission over long lines. He attacked the problem by making a study of the line itself with respect to the line constants, namely, resistance, inductance, capacity and conductance. He found that as an electric wave progressed along the line from the transmitting end to the receiving end various influences caused the wave to become attenuated and distorted so that by the time it reached the receiving apparatus the wave had an entirely different amplitude and shape than the original wave. The consequence of this was that the speech heard at the receiving end was

entirely different in volume and quality from the speech transmitted. Now by changing the line constants by means of auxiliary inductances he found that the telephone waves could be transmitted with better efficiency. The method used for increasing the efficiency of wave transmission is termed "loading" and consists of spacing coils of known inductance at certain points along the line. This method is used almost universally in the United States and in many foreign countries.

No doubt the invention of the loading coil has done more toward extending the limit of telephone transmission than has any other invention. However, it has greatly increased the initial expense of constructing long distance telephone lines and furthermore increases the expense of maintenance.

This thesis was conducted with the purpose in view of studying the current and voltage relations in long-distance telephone lines and, if possible, of effecting an improvement in the transmission of speech over long lines, by means of additional terminal apparatus.



Diagrammatical Telephone Line.

Fig. 1

CHAPTER II.

Theory of Long Distance Transmission.

Let Fig.1 respresent, diagrammatically, a long-distance telephone line with a telephone set bridged across the end of the line. Every line has a certain resistance and inductance. There is also a capacity effect and conductance between the two sides of the line. In the sketch the resistance, inductance, capacity and conductance are shown grouped at points along the line. However in the real line the resistance, inductance, capacity and conductance are uniformly distributed over the entire length of the line.

Notation.

b = susceptance per unit loop distance - $2\pi fc$
C = capacity " " " "
q = conductance " " " "
f = frequency " " " "
L - inductance " " " "
R = resistance " " " "
x - inductive reactance per unit loop distance
y = admittance " " " "
z = impedance " " " "

On the assumption that the current and voltage in a telephone line are sine waves let them be represented by $\underset{\cdot}{I}$ and $\underset{\cdot}{E}$ respectively at any point in the line. Then if distance is measured toward the transmitting end of the line and power is flowing in the opposite direction the current at a point closer to the generating end by a distance, Δl has a greater value than $\underset{\cdot}{I}$ by an

amount equal to the vector sum of the charging current and leakage current. Similarly the voltage at this point is greater than $\underset{\cdot}{E}$ by an amount equal to the impedance drop over the distance Δl. Let $\Delta\underset{\cdot}{I}$ and $\Delta\underset{\cdot}{E}$ represent the increments in current and voltage respectively over the distance Δl. Thus using the vector method of notation the following equations can be set up,-

$$\Delta\underset{\cdot}{I} = \underset{\cdot}{E}(q + jb)\Delta l$$

$$\Delta\underset{\cdot}{E} = \underset{\cdot}{I}(R + jx)\Delta l$$

Dividing by Δl

$$\frac{\Delta\underset{\cdot}{I}}{\Delta l} = \underset{\cdot}{E}(q + jb)$$

$$\frac{\Delta\underset{\cdot}{E}}{\Delta l} = \underset{\cdot}{I}(R + jx)$$

Now if Δl is taken infinitely small the above equations reduce to the form,

$$\frac{d\underset{\cdot}{I}}{dl} = \underset{\cdot}{E}(q + jb) \qquad (1)$$

$$\frac{d\underset{\cdot}{E}}{dl} = \underset{\cdot}{I}(R + jx) \qquad (2)$$

Taking the second derivative with respect to l the following equations are obtained,-

$$\frac{d^2\underset{\cdot}{I}}{dl^2} = \frac{d\underset{\cdot}{E}}{dl}(q + jb) \qquad (3)$$

$$\frac{d^2\underset{\cdot}{E}}{dl^2} = \frac{d\underset{\cdot}{I}}{dl}(R + jx) \qquad (4)$$

Substituting equation No.2 in No.3 for $\frac{d\underset{\cdot}{E}}{dl}$ the equation for $\frac{d^2\underset{\cdot}{I}}{dl}$ becomes,

$$\frac{d^2\underset{\cdot}{I}}{dl^2} = \underset{\cdot}{I}\,(R + jx)(q + jb) \qquad (5)$$

Similarly substituting No.1 in No.4,-

$$\frac{d^2\underset{\cdot}{E}}{dl^2} = \underset{\cdot}{E}\,(q + jb)(R + jx) \qquad (6)$$

Now (q + jb) is the vector expression for admittance and (R + jx) is the vector expression for impedance thus

$$\underset{\cdot}{y} = q + jb$$

$$\underset{\cdot}{z} = R + jx$$

Substituting in equations Nos. 5 and 6

$$\frac{d^2\underset{\cdot}{I}}{dl} = \underset{\cdot}{I}\underset{\cdot}{z}\underset{\cdot}{y} \qquad (7)$$

$$\frac{d^2\underset{\cdot}{E}}{dl} = \underset{\cdot}{E}\underset{\cdot}{z}\underset{\cdot}{y} \qquad (8)$$

Equations No.s 7 and 8 are first degree equations of the second order and are solved by letting

$$\underset{\cdot}{I} = Ae^{-Ml} \qquad (9)$$

where A is an arbitrary constant, e is the base of natural logarithms and M is a constant.

$$\frac{d\underset{\cdot}{I}}{dl} = A(-M)e^{-Ml}$$

$$\frac{d^2\underset{\cdot}{I}}{dl} = AM^2e^{-Ml} \qquad (10)$$

Substituting No.s 9 and 10 in No. 7 the following equation results

$$AM^2e^{-Ml} = Ae^{-Ml}\underset{\cdot}{z}\underset{\cdot}{y}$$

$$M^2 = \underset{\cdot}{z}\underset{\cdot}{y}$$

$$M = \pm\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}} \qquad (11)$$

Thus the auxiliary equation has two roots $+\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}$ and $-\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}$ and $\underset{\cdot}{I} = A_1e^{+\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l}$ and $\underset{\cdot}{I} = A_2e^{-\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l}$ are particular solutions.

Then the general solution is,

$$\underset{\cdot}{I} = A_1 e^{\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l} + A_2 e^{-\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l} \qquad (12)$$

Similarly it can be shown that the equation for $\underset{\cdot}{E}$ is,

$$\underset{\cdot}{E} = B_1 e^{\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l} + B_2 e^{-\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l} \qquad (13)$$

It is now necessary to evaluate the constants A_1, A_2, B_1 and B_2. Refering to equation No. 1

$$\underset{\cdot}{E} = \frac{d\underset{\cdot}{I}}{dl}\,\frac{1}{(g + jb)} = \frac{d\underset{\cdot}{I}}{dl}\,\frac{1}{\underset{\cdot}{y}} \qquad (14)$$

Differentiating equation No. 12 with respect to l,

$$\frac{d\underset{\cdot}{I}}{dl} = A_1\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\;e^{-\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l} - A_2\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\;e^{-\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l} \qquad (15)$$

Substituting equation No. 15 in No. 14

$$\underset{\cdot}{E} = \frac{\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}}{\underset{\cdot}{y}}\left(A_1 e^{\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l} - A_2 e^{-\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}\,l}\right) \qquad (16)$$

Now when l = 0, that is at the receiving end of the line, the current is $\underset{\cdot}{I}_0$ and the voltage is $\underset{\cdot}{E}_0$ the values of $\underset{\cdot}{I}_0$ and $\underset{\cdot}{E}_0$ depending on the constants of the receiving apparatus. Thus from equations No.s 12 and 16 the following equations are obtained when l = 0

$$\underset{\cdot}{I}_0 = A_1 + A_2 \qquad (17)$$

$$\underset{\cdot}{E}_0 = \frac{\sqrt{\underset{\cdot}{z}\underset{\cdot}{y}}}{\underset{\cdot}{y}}(A_1 - A_2) \qquad (18)$$

Solving the above equations for A_1 and A_2 it is found that,

$$A_1 = \tfrac{1}{2}\left(\underset{\cdot}{E}_0\sqrt{\frac{\underset{\cdot}{y}}{\underset{\cdot}{z}}} + \underset{\cdot}{I}_0\right) \qquad (19)$$

$$A_2 = \tfrac{1}{2}\left(\underset{\cdot}{I}_0 - \underset{\cdot}{E}_0\sqrt{\frac{\underset{\cdot}{y}}{\underset{\cdot}{z}}}\right) \qquad (20)$$

By a similar method it is found that,

$$B_1 = \tfrac{1}{2}\left(\underset{\cdot}{I}_0\sqrt{\frac{\underset{\cdot}{z}}{\underset{\cdot}{y}}} + \underset{\cdot}{E}_0\right) \qquad (21)$$

$$B_2 = \tfrac{1}{2}\left(\underset{\cdot}{E}_0 - \underset{\cdot}{I}_0\sqrt{\frac{\underset{\cdot}{z}}{\underset{\cdot}{y}}}\right) \qquad (22)$$

Now substituting equations Nos. 19 and 20 in No. 12

$$\underset{\cdot}{I} = \tfrac{1}{2}(\underset{\cdot}{I}_0 + \underset{\cdot}{E}_0\sqrt{\frac{\underset{\cdot}{Y}}{\underset{\cdot}{Z}}})e^{\sqrt{\underset{\cdot}{Z}\underset{\cdot}{Y}}\,l} + \tfrac{1}{2}(\underset{\cdot}{I}_0 - \underset{\cdot}{E}_0\sqrt{\frac{\underset{\cdot}{Y}}{\underset{\cdot}{Z}}})e^{-\sqrt{\underset{\cdot}{Z}\underset{\cdot}{Y}}\,l} \qquad (23)$$

and substituting Nos. 21 and 22 in No. 13,

$$\underset{\cdot}{E} = \tfrac{1}{2}(\underset{\cdot}{E}_0 + \underset{\cdot}{I}_0\sqrt{\frac{\underset{\cdot}{Z}}{\underset{\cdot}{Y}}})e^{\sqrt{\underset{\cdot}{Z}\underset{\cdot}{Y}}\,l} + \tfrac{1}{2}(\underset{\cdot}{E}_0 - \underset{\cdot}{I}_0\sqrt{\frac{\underset{\cdot}{Z}}{\underset{\cdot}{Y}}})e^{-\sqrt{\underset{\cdot}{Z}\underset{\cdot}{Y}}\,l} \qquad (24)$$

These last two equations give the current and voltage at any point a distance l from the receiving end of the line, but the equations in their present form are symbolic expressions and do not permit of calculation because of the exponential factors. So to make the equations of practical value the exponential terms must be reduced to a form which will permit of calculation.

Let

$$e^{\sqrt{\underset{\cdot}{Z}\underset{\cdot}{Y}}\,l} = e^{(\alpha + j\beta)l}$$

Then

$$\alpha + j\beta = \sqrt{\underset{\cdot}{Z}\underset{\cdot}{Y}} \qquad (25)$$

$$\alpha^2 + 2j\alpha\beta - \beta^2 = \underset{\cdot}{Z}\underset{\cdot}{Y} \qquad (26)$$

$$\underset{\cdot}{Z} = R + jx$$

$$\underset{\cdot}{Y} = q + jb$$

Therefore,

$$\underset{\cdot}{Z}\underset{\cdot}{Y} = (R + jx)(q + jb)$$

$$\underset{\cdot}{Z}\underset{\cdot}{Y} = Rq + jRb + jxq - xb \qquad (27)$$

Substituting No. 27 in No. 26,

$$\alpha^2 + j2\alpha\beta - \beta^2 = Rq - xb + jRb + jxq$$

Equating the real terms and the imaginary terms,

$$\alpha^2 - \beta^2 = Rq - xb \qquad (28)$$

$$2\alpha\beta = + Rb + xq \qquad (29)$$

Equations Nos. 28 and 29 are two simultaneous equations by means of which values for α and β can be obtained thus,-

Squaring both equations,

$$\alpha^4 - 2\alpha^2\beta^2 + \beta^4 = (Rq - xb)^2$$

$$4\alpha^2\beta^2 = (+Rb + xq)^2$$

and taking the sum

$$\alpha^4 + 2\alpha^2\beta^2 + \beta^4 = (Rq - xb)^2 + (+Rb + xg)^2 \qquad (30)$$

Expanding the right hand term of equation No. 30 there results,

$$(Rq)^2 - 2Rqxb + (xb)^2 + (Rb)^2 + 2Rbxq + (xq)^2 =$$

$$(Rq)^2 + (xb)^2 + (Rb)^2 + (xq)^2 =$$

$$R^2(q^2 + b^2) + x^2(b^2 + q^2) =$$

$$(R^2 + x^2)(q^2 + b^2)$$

Now $(R^2 + x^2) = z^2$ and $(q^2 + b^2) = y^2$ when z and y are absolute values of impedance and admittance and not vector quantities. Substituting in equation No. 30,

$$\alpha^4 + 2\alpha^2\beta^2 + \beta^4 = z^2y^2 \qquad (31)$$

$(\alpha^4 + 2\alpha^2\beta^2 + \beta^4)$ is an exact square, so extracting the square root of both sides of equations No. 31 there obtains

$$\alpha^2 + \beta^2 = zy \qquad (32)$$

Now by means of equations Nos. 32 and 28 values of α and β can be obtained,

$$\alpha^2 + \beta^2 = zy$$

$$\alpha^2 - \beta^2 = Rq - xb$$

$$\alpha = \sqrt{\tfrac{1}{2}(zy + Rq + xb)} \qquad (33)$$

$$\beta = \sqrt{\tfrac{1}{2}(zy - Rq + xb)} \qquad (34)$$

The equations for current and voltage at a distance l from the receiving end of the line are respectively,

$$\underset{\cdot}{I} = \tfrac{1}{2}\left(\underset{\cdot}{I}_0 + \underset{\cdot}{E}_0\sqrt{\frac{\underset{\cdot}{y}}{\underset{\cdot}{z}}}\right)e^{(\alpha + j\beta)l} + \tfrac{1}{2}\left(\underset{\cdot}{I}_0 - \underset{\cdot}{E}_0\sqrt{\frac{\underset{\cdot}{y}}{\underset{\cdot}{z}}}\right)e^{-(\alpha + j\beta)l} \qquad (35)$$

$$\underset{\cdot}{E} = \tfrac{1}{2}(\underset{\cdot}{E}_o + \underset{\cdot}{I}_o\sqrt{\tfrac{\underset{\cdot}{z}}{\underset{\cdot}{y}}})e^{(\alpha + j\beta)l} + \tfrac{1}{2}(\underset{\cdot}{E}_o - \underset{\cdot}{I}_o\sqrt{\tfrac{\underset{\cdot}{z}}{\underset{\cdot}{y}}})e^{-(\alpha + j\beta)l} \quad (36)$$

It now remains to obtain a solution of $e^{(\alpha + j\beta)l}$ which does not contain complex exponents. It can be shown that,

$$e^{(\alpha l + j\beta l)} = e^{\alpha l}\, e^{j\beta l} \quad (37)$$

$e^{\alpha l}$ is a real number, but $e^{j\beta l}$ has no arithmetical sense, so another form must be sought which will permit of calculation.

Expanding $e^{j\beta l}$ by Maclauren's formula it becomes,

$$e^{j\beta l} = 1 + \frac{j\beta l}{\lfloor\!\underline{1}} + \frac{(j\beta l)^2}{\lfloor\!\underline{2}} + \frac{(j\beta l)^3}{\lfloor\!\underline{3}} + \frac{(j\beta l)^4}{\lfloor\!\underline{4}} ---- \quad (38)$$

Now,

$$j = j$$

$$j^2 = -1$$

$$j^3 = -j$$

$$j^4 = 1$$

Substituting in the above

$$e^{j\beta l} = 1 + \frac{j\beta l}{\lfloor\!\underline{1}} - \frac{(\beta l)^2}{\lfloor\!\underline{2}} - \frac{j(\beta l)^3}{\lfloor\!\underline{3}} + \frac{(\beta l)^4}{\lfloor\!\underline{4}} ----- \quad (39)$$

Expanding $\cos\beta l$ and $\sin\beta l$ by Maclauren's theorem,

$$\cos\beta l = 1 - \frac{(\beta l)^2}{\lfloor\!\underline{2}} + \frac{(\beta l)^4}{\lfloor\!\underline{4}} - \frac{(\beta l)^6}{\lfloor\!\underline{6}} ------- \quad (40)$$

$$\sin\beta l = \frac{\beta l}{\lfloor\!\underline{1}} - \frac{(\beta l)^3}{\lfloor\!\underline{3}} + \frac{(\beta l)^5}{\lfloor\!\underline{5}} --------- \quad (41)$$

Multipying both sides of equation No. 41 by j

$$j\sin\beta l = \frac{j\beta l}{\lfloor\!\underline{1}} - \frac{j(\beta l)^3}{\lfloor\!\underline{3}} + \frac{j(\beta l)^5}{\lfloor\!\underline{5}} -------- \quad (42)$$

Taking the sum of equations Nos. 40 and 42,

$$\cos\beta l + j\sin\beta = 1 + \frac{j\beta l}{\lfloor\!\underline{1}} - \frac{(\beta l)^2}{\lfloor\!\underline{2}} - \frac{j(\beta l)^3}{\lfloor\!\underline{3}} + \frac{(\beta l)^4}{\lfloor\!\underline{4}} -- \quad (43)$$

It is seen that the right members of equations Nos. 43 and 39 are equal. Therefore,

$$e^{j\beta l} = \cos\beta l + j\sin\beta l \qquad (44)$$

Similarly it can be proved that

$$e^{-j\beta l} = \cos\beta l + j\sin\beta l \qquad (45)$$

Therefore,

$$e^{\alpha l + j\beta l} = e^{\alpha l}(\cos\beta l + j\sin\beta l) \qquad (46)$$

and

$$e^{-\alpha l - j\beta l} = e^{-\alpha l}(\cos\beta l - j\sin\beta l) \qquad (47)$$

Substituting equations Nos. 46 and 47 in Nos. 35 and 36

$$I = \tfrac{1}{2}\left(I_0 + E_0\sqrt{\tfrac{Y}{Z}}\right)e^{\alpha l}(\cos\beta l + j\sin\beta l) + \tfrac{1}{2}\left(I_0 - E_0\sqrt{\tfrac{Y}{Z}}\right)e^{-\alpha l}(\cos\beta l - j\sin\beta l) \qquad (48)$$

$$E = \tfrac{1}{2}\left(E_0 + I_0\sqrt{\tfrac{Z}{Y}}\right)e^{\alpha l}(\cos\beta l + j\sin\beta l) + \tfrac{1}{2}\left(E_0 - I_0\sqrt{\tfrac{Z}{Y}}\right)e^{-\alpha l}(\cos\beta l - j\sin\beta l) \qquad (49)$$

To simplify the equations still further it is necessary to determine $\sqrt{\frac{Z}{Y}}$ in terms of constants h and K

Let

$$\sqrt{\frac{Z}{Y}} = h + jK \qquad (50)$$

Then

$$\frac{Z}{Y} = h^2 + j2hK - K^2 \qquad (51)$$

$$\frac{Z}{Y} = \frac{R + jx}{q + jb} \qquad (52)$$

Rationalizing

$$\frac{R + jx}{q + jb} = \frac{(R + jx)}{(q + jb)}\,\frac{q - jb}{q - jb} = \frac{Rq + jqx - jRb + xb}{q^2 + b^2} \qquad (53)$$

$$q^2 + b^2 = Y^2 \qquad (54)$$

Therefore substituting

$$h^2 + j2hK - k^2 = \frac{Rq + jqx - jRb + xb}{y^2} \qquad (55)$$

Equating the real terms and the imaginary terms,

$$h^2 - K^2 = \frac{Rq + xb}{y^2} \qquad (56)$$

$$2hK = \frac{qx - Rb}{y^2} \qquad (57)$$

Squaring equations Nos. 56 and 57

$$(h^4 - 2h^2K^2 + K^4) = \frac{(Rq)^2 + 2\,Rqxb + (xb)^2}{y^4} \qquad (58)$$

$$4h^2K^2 = \frac{(qx)^2 - 2Rqxb + (Rb)^2}{y^4} \qquad (59)$$

Taking the sum,

$$h^4 + 2h^2K^2 + K^4 = \frac{(Rq)^2 + (xb)^2 + (qx)^2 + (Rb)^2}{y^4} \qquad (60)$$

The right hand member of this equation reduces to,

$$\frac{(Rq)^2 + (xb)^2 + (qx)^2 + (Rb)^2}{y^4} = \frac{(R^2 + x^2)(q^2 + b^2)}{y^4} = \frac{z^2y^2}{y^4} \qquad (61)$$

Substituting No. 61 in No. 60

$$h^4 + 2h^2K^2 + K^4 = \frac{z^2y^2}{y^4} \qquad (62)$$

Taking the square root of both sides of No. 62

$$h^2 + K^2 = \frac{zy}{y^2} \qquad (63)$$

Nowsolving equations Nos. 56 and 63 simultaneously it is found that,

$$h = \sqrt{\frac{1}{2}\,\frac{(Rq + xb + zy)}{y^2}} \qquad (64)$$

$$K = \sqrt{\frac{1}{2}\,\frac{(-Rq - xb + zy)}{y^2}} \qquad (65)$$

Now the equations for determining the vector expressions of current and voltage at any point in a line at a distance l from the receiving end of the line are given in their simplest form as follows,-

$$\underset{\cdot}{I} = \tfrac{1}{2}\left(\underset{\cdot}{I}_0 + \frac{\underset{\cdot}{E}_0}{h + jK}\right)e^{\alpha l}(\cos\beta l + j\sin\beta l) + \tfrac{1}{2}\left(\underset{\cdot}{I}_0 \ominus \frac{\underset{\cdot}{E}_0}{h + jK}\right)$$

$$e^{-\alpha l}(\cos\beta l - j\sin\beta l) \qquad (66)$$

$$\underset{\cdot}{E} = \left\{\tfrac{1}{2}\underset{\cdot}{E}_0 + \underset{\cdot}{I}_0(h + jK)\right\}e^{\alpha l}(\cos\beta l + j\sin\beta l) +$$

$$\tfrac{1}{2}\left\{\underset{\cdot}{E}_0 - \underset{\cdot}{I}_0(h + jK)\right\}e^{-\alpha l}(\cos\beta l - j\sin\beta l) \qquad (67)$$

A summary of the equations used in determining the constants of equations Nos. 66 and 67

$$\alpha = \sqrt{\tfrac{1}{2}(zy + Rq - xb)} \qquad (68)$$

$$\beta = \sqrt{\tfrac{1}{2}(zy - Rq + xb)} \qquad (69)$$

$$h = \frac{1}{y}\sqrt{\tfrac{1}{2}(zy + Rq + xb)} \qquad (70)$$

$$K = \frac{1}{y}\sqrt{\tfrac{1}{2}(zy - Rq - xh)} \qquad (71)$$

Equations Nos. 66 and 67 might be reduced to a slightly more simple form by means of the following substitution,-

If the line terminates in a telephone set as shown in Fig. 1 then the voltage at that end of the line is equal to the current multiplied by the impedance of the telephone set, thus

$$\underset{\cdot}{E}_0 = \underset{\cdot}{I}_0(r_1 + jx_1) \qquad (72)$$

where r_1 and x_1 are the resistance and inductive reactance respectively of the telephone set. Then substituting in the equa-

tions for the current and voltage the following equations are obtained,

$$\underset{\cdot}{I} = \tfrac{1}{2}\left(\underset{\cdot}{I}_0 + \underset{\cdot}{I}_0\frac{(r_1 + jx_1)}{(h + jK)}\right)e^{\alpha l}(\cos\beta l + j\sin\beta l) +$$

$$\tfrac{1}{2}\left(\underset{\cdot}{I}_0 - \underset{\cdot}{I}_0\frac{(r_1 + jx_1)}{(h + jK)}\right)e^{-\alpha l}(\cos\beta l - j\sin\beta l) \qquad (73)$$

$$\underset{\cdot}{E} = \tfrac{1}{2}(\underset{\cdot}{I}_0\left\{(r_1 + jx_1) + (h + jK)\right\} e^{\alpha l}(\cos\beta l + j\sin\beta l) +$$

$$\tfrac{1}{2}\,\underset{\cdot}{I}_0\left\{(r_1 + jx_1) - (h + jK)\right\} e^{-\alpha l}(\cos\beta l - j\sin\beta l) \qquad (74)$$

An examination of equations Nos. 73 and 74 will bring out several interesting features. It is seen that the current in the line consists of two components, which will be called the "incoming" current and the "reflected" wave of current. The expression "incoming wave of current" means that the power is flowing toward the receiving apparatus and "reflected wave of current" means the power is flowing from the receiving end of the line back toward the generating end. It is also seen that the voltage consists of two components, one which causes the "incoming wave of current" to flow and the other causes the "reflected wave of current" to flow.

Thus the current and voltage equations may be written,

$$\underset{\cdot}{I} = \underset{\cdot}{I}' + \underset{\cdot}{I}'' \qquad (75)$$

$$\underset{\cdot}{E} = \underset{\cdot}{E}' + \underset{\cdot}{E}'' \qquad (76)$$

where $\underset{\cdot}{I}'$ and $\underset{\cdot}{I}''$ are the incoming and reflected waves of current respectively and $\underset{\cdot}{E}'$ and $\underset{\cdot}{E}''$ are the voltages which cause the

incoming and reflected currents to flow.

$$I' = \frac{I_0}{2}\left(1 + \frac{(r_1 + jx_1)}{h + jK}\right) e^{\alpha l}(\cos\beta l + j\sin\beta l) \quad (77)$$

$$I'' = \frac{I_0}{2}\left(1 - \frac{(r_1 + jx_1)}{(h + jK)}\right) e^{-\alpha l}(\cos\beta l - j\sin\beta l) \quad (78)$$

$$E' = \frac{I_0}{2}\left\{(r_1 + jx_1) + (h + jK)\right\} e^{\alpha l}(\cos\beta l + j\sin\beta l) \quad (79)$$

$$E'' = \frac{I_0}{2}\left\{(r_1 + jx_1) - (h + jK)\right\} e^{-\alpha l}(\cos\beta l - j\sin\beta l) \quad (80)$$

It is seen that the expression for the incoming wave of current, No. 77, consists of three factors, a constant, the variable exponential factor and the expression, $(\cos\beta l + j\sin\beta l)$. Now the value of the constant,

$$\frac{I_0}{2}\left\{1 + \frac{(r_1 + jx_1)}{(h + jK)}\right\}$$

depends on the constants of the receiving apparatus, and on the line constants. The exponential factor, however, is the one which determines the value of the incoming current at any point in the line. As l, the distance from the receiving end of the line, increases the term $e^{\alpha l}$ increases. Hence the incoming wave of current becomes greater, the greater the distance from the end of the line. The rate at which the incoming wave of current increases depends on the value of the constant α. Consequently this constant is called the "attenuation constant" of the line, and as shown in equation No. 33

$$= \sqrt{\tfrac{1}{2}(zy + Rq - xb)}$$

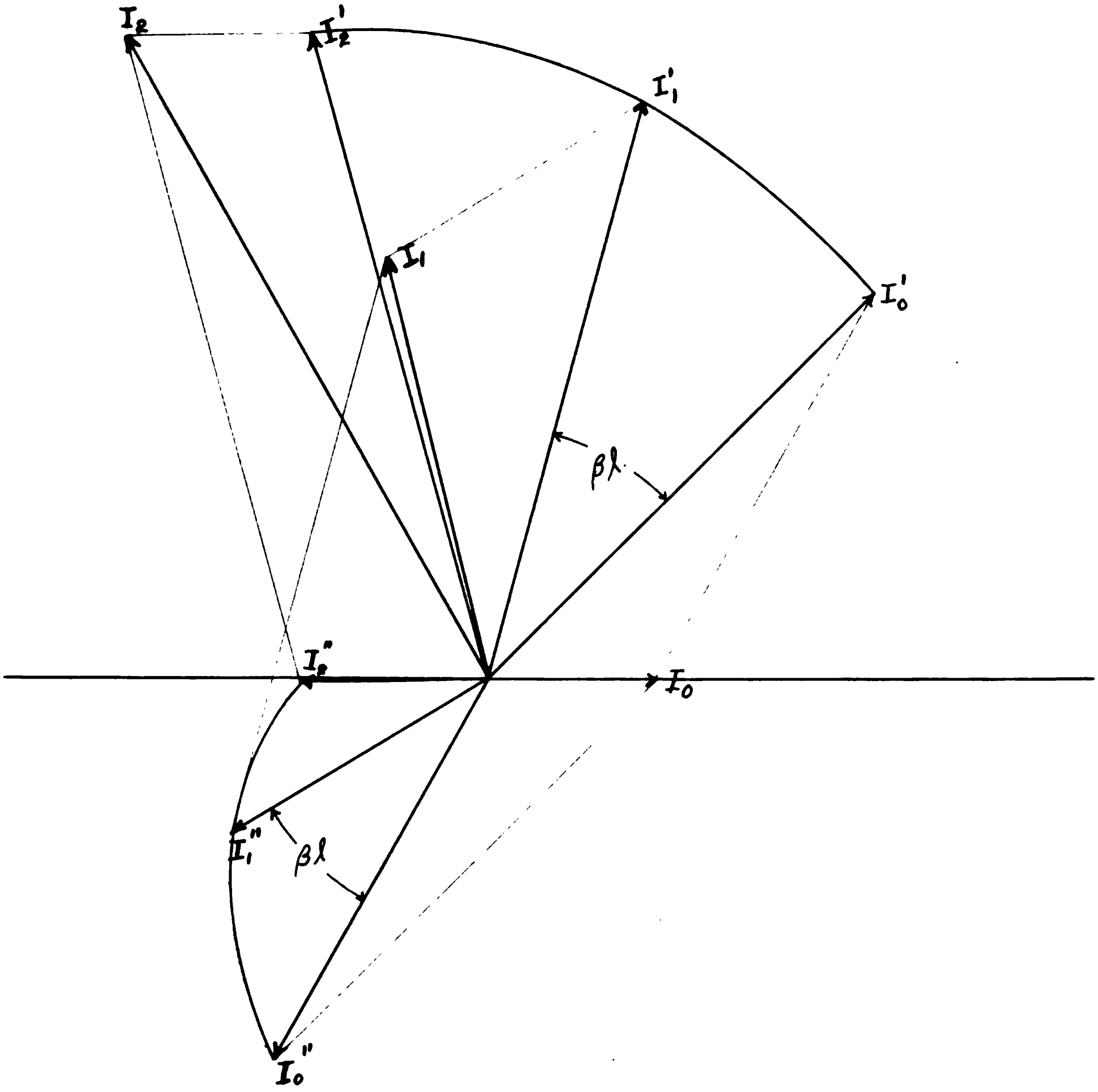
The factor $(\cos\beta l + j\sin\beta l)$ does not in any way affect the absolute value of the incoming wave of current, but simply determines the position of the current vector, with reference to the current vector for l equals to zero. $(\cos\beta l + j\sin\beta l)$ as a multiplier, revolves the vector in a counter clockwise direction and by the time βl becomes 2π the incoming wave of current has passed through one wave length. Hence is called the "wave length constant" because the value of β determines the length of one wave. Thus if λ represents wave length,

$$\lambda = \frac{2\pi}{\beta} \qquad (81)$$

What has just been written in regard to the incoming wave of current might be repeated for the reflected wave of current except for a few modifications. The equation for the reflected wave of current is shown in No. 78. Here it is seen that the constant term is less in magnitude than that of the incoming wave of current. The exponential term enters in the denominator and hence as the distance from the receiving end of the line increases the reflected wave of current decreases. The factor, $(\cos\beta l - j\sin\beta l)$ revolves the reflected current vector in a clockwise direction opposite to the direction of rotation of the incoming current vector.

A clearer idea of the equation showing current at any point in the line can be gained by plotting a vector diagram, as shown in Fig. 21 At the receiving end of the line, l is equal to

Fig. 2.



Vector Relations of Currents.

zero, and the incoming wave of current is equal to the constant term,

$$\underset{\cdot}{I}' = \frac{I_o}{2}\left\{1 + \frac{(r_1 + jx_1)}{(h + jK)}\right\}$$

The second component of the current, reflected wave of current, at the end of the line is equal to the constant term,

$$\underset{\cdot}{I}'' = \frac{I_o}{2}\left\{1 - \frac{(r_1 + jx_1)}{(h + jK)}\right\}$$

In the diagram the incoming and reflected waves of current at the receiving end of the line are shown by the vectors $\underset{\cdot}{I}_o'$ and $\underset{\cdot}{I}_o''$ respectively and the total current, $\underset{\cdot}{I}_o'$ at that same end of the line is shown to be the vector sum of $\underset{\cdot}{I}_o'$ and $\underset{\cdot}{I}_o''$.

Now suppose it is desired to find the current at some point, (1), in the line a distance 1 from the receiving end. Considering first the incoming wave of current, it is seen from equation No. 77 that the value of this component at point (1) is the product of the constant term and $e^{\alpha l}$. So taking the product of the current vector $\underset{\cdot}{I}_o'$ and $e^{\alpha l}$, the length of the incoming current vector at point (1) is determined. The direction of this vector is determined by the term $(\cos \beta l + j \sin \beta l)$. Thus multiplying the vector $\underset{\cdot}{I}_o'$ by $(\cos \beta l + j \sin \beta l)$ merely revolves that vector through an angle βl in a counter clockwise direction. Now the length and direction of the incoming current vector at point (1) has been determined and is shown in Fig. 2 as $\underset{\cdot}{I}_1'$.

Similarly the magnitude and direction of the reflected current vector at point (1) is determined by taking the product

of $\underset{\cdot}{I}_0''$, $e^{-\alpha l}$ and the term $(\cos \beta l - j \sin \beta l)$. It is seen that the term $e^{-\alpha l}$ causes the magnitude of the reflected wave of current to decrease and the term $(\cos \beta l - j \sin \beta l)$ revolves the vector through an angle βl in a clockwise direction. The reflected current vector at point (1) is then shown by the vector $\underset{\cdot}{I}_1''$. The total current in the line at point (1) is then the vector sum of $\underset{\cdot}{I}_1'$ and $\underset{\cdot}{I}_1''$ and is shown as $\underset{\cdot}{I}_1$. Likewise the current at other points in the line can be determined.

A similar diagram might be constructed which would show the vector relations of the voltage components but all that has been said in regard to the current diagram holds good for a voltage diagram.

Now in Fig. 2 if several points are chosen and curves drawn through the ends of the vectors it is seen that the incoming wave of current increases as the distance from the receiving end of the line increases or rather, as the distance out from the generating end becomes less, and it is seen that the reflected wave of current becomes less. It must be born in mind that these curves are really three dimension curves plotted in two dimensions. Thus any point on the curve shows the magnitudes of the current, the phase relation of the current with reference to the current at the end of the line and the distance from the receiving end of the line. If the curve could be plotted in three dimensions, it would resemble very closely the curve of a conical spiral spring. These current and voltage curves will probably be better understood by refering to the discussion of the actual curves for the line.

Thus far the two components of the current have been referred to as the incoming and reflected waves of current. It has also been said that the incoming wave of current is that component of the total current wave which is flowing from the generating end to the receiving end of the line and that the reflected wave of current is that component which flows from the receiving end toward the generating end of the line. Furthermore it has been shown that the voltage consists of two components, one of which causes the incoming current wave to flow and the other of which causes the reflected current wave to flow. However, as yet nothing has been given which proves that the reflected current wave is flowing back toward the generating end and therefore represents lost power. In the derivation of the current and voltage equations it was assumed that power was positive when flowing from the generating end of the line to the receiving end. Hence power flowing from the receiving end of the line back toward the generating end will have a negative sign. Therefore, to prove that the incoming current wave is flowing from the generating end of the line and that the reflected current wave is flowing toward the generating end it must be shown that the incoming current wave represents positive power and that the reflected current wave represents negative power.

Now if the expression for the current of the incoming wave is written,

$$\underset{\cdot}{I}' = i_1 + ji_2$$

and the voltage causing the incoming wave of current to flow is expressed

$$\underset{\cdot}{E}' = e_1 + je_2$$

then the power will be

$$P' = e_1 i_1 + e_2 i_2$$

where P' is the power represented by the incoming current wave.

Similarly the reflected current wave and corresponding voltage can be expressed,

$$\underset{\cdot}{I}'' - i_3 + ji_4$$

$$\underset{\cdot}{E}'' = e_3 + je_4$$

and the power represented by the reflected current wave will be

$$P'' = i_5 e_3 + i_4 e_4$$

Now the proposition resolves itself into one of proving that P' is a positive quantity and that P'' is a negative quantity.

Referring to equations Nos. 77 and 79

$$\underset{\cdot}{I}' = \frac{I_o}{2}\left\{1 + \frac{(r_1 + jx_1)}{(h + jK)}\right\} e^{\alpha l}(\cos\beta l + j\sin\beta l)$$

$$\underset{\cdot}{E}' - \frac{I_o}{2}\left\{(r_1 + jx_1) + (h + jK)\right\} e^{\alpha l}(\cos\beta l + j\sin\beta l)$$

the factors $e^{\alpha l}$ and $(\cos\beta l + j\sin\beta l)$ do not in any way affect the sign of the power, so can be dropped. $(r_1 + jx_1)$ and $(\overset{h}{L} + JK)$ are vector quantities. Hence the following substitutions can be made.

Let $(r_1 + jx_1) - m(\cos\phi + j\sin\phi)$ (84)

$(h + jK) - n(\cos\theta + j\sin\theta)$ (85)

Then $\dfrac{1}{(h + jK)} = \dfrac{1}{n}(\cos\theta - j\sin\theta)$

Substituting in equations

$$\underset{\cdot}{I}' = \left\{1 + \frac{m}{n}(\cos\phi + j\sin\phi)(\cos\theta - j\sin\theta)\right\}\frac{I_o}{2}$$

$$\underset{\cdot}{E}' = \left\{m(\cos\phi + j\sin\phi) + n(\cos\theta + j\sin\theta)\right\}\frac{I_o}{2}$$

$$\underset{\cdot}{I}' = \left\{(1 + \frac{m}{n}\cos\phi\cos\theta + \frac{m}{n}\sin\phi\sin\theta) + j(\frac{m}{n}\sin\phi\cos\theta - \frac{m}{n}\sin\theta\cos\phi)\right\}\frac{I_o}{2}$$

$$\underset{\cdot}{E}' = \left\{(m\cos\phi + n\cos\theta) + j(m\sin\phi + n\sin\theta\right\}\frac{I_o}{2}$$

Now as stated previously the power is equal to the sum of the products of the real terms and the j^2 terms.

Therefore,

$$P' = \frac{I_o^2}{4}\left\{(m\cos\phi + n\cos\theta + \frac{m^2}{n}\cos^2\phi\cos\theta + m\cos\phi\cos^2\theta + \frac{m^2}{n}\sin\phi\cos\phi\sin\theta + m\sin\phi\sin\theta\cos\theta) + (\frac{m^2}{n}\sin^2\phi\cos\theta + m\sin\phi\cos\theta\sin\theta - \frac{m^2}{n}\sin\theta\cos\phi\sin\phi - m\sin^2\theta\cos\phi)\right\}$$

$$P' = \frac{I_o^2}{4}\left\{m\cos\phi + n\cos\theta + \frac{m^2}{n}\cos\theta + m\cos\phi(\cos^2\theta - \sin^2\theta) + 2m\sin\theta\cos\theta\sin\phi\right\}$$

$$P' = \frac{I_o^2}{4}\left\{n\cos\theta + \frac{m^2}{n}\cos\theta + m\cos\phi\cos 2\theta + m\sin\phi\sin 2\theta + m\cos\phi\right\}$$

Now $m(\cos \phi \cos 2\theta + \sin \phi \sin 2\theta) = m(\cos(\phi - 2\theta)$

and $m(\cos(\phi - 2\theta) + m\cos\phi = 2m\cos(\phi - \theta) \cos \theta$

Therefore,

$$P' = \frac{I^2}{4}\left(\left\{n + \frac{m^2}{n} + 2m\cos(\phi - \theta)\right\} \cos \theta\right)$$

Now the factor $\frac{I^2}{4}$ does not in any way affect the sign of P' so it remains to be shown that the expression $\left(\left\{n + \frac{m^2}{n} + 2m\cos(\phi - \theta)\right\}\cos\theta\right)$ is a positive quantity. For this to be true the angles $(\phi - \theta)$ and θ must not be greater than $+ 90^\circ$ nor less than $- 90^\circ$.

Now from equation No. 84 it is seen that

$$\phi = \tan^{-1} \frac{x_1}{r_1}$$

r_1 must always be a positive quantity but x_1 may be either positive or negative. Hence the extreme limits for ϕ are $+90^\circ$ or -90°.

From equation No. 85

$$\theta = \tan^{-1} \frac{K}{h}$$

From the manner in which h and K were derived they must both be positive quantities. Hence θ may vary from 0° to $+90^\circ$.

Now the $\cos \theta$ must always be positive but $\cos(\phi - \theta)$ may be a negative quantity. Taking the extreme case when $\cos(\phi - \theta)$ is equal to -1, the expression

$$n + \frac{m^2}{n} + 2m\cos(\phi - \theta)$$

becomes,

$$n + \frac{m^2}{n} - 2m = \frac{(n - m)^2}{n}$$

which is a positive quantity. Therefore, since all the terms of

the power equation must be positive it follows that the power represented by the incoming current must always be positive and hence the incoming wave of current is flowing toward the receiving end of the line.

By a similar method it can be shown that

$$P'' = \frac{\underset{\cdot}{I}_o^2}{4}\left(\left\{-n - \frac{m^2}{n} + 2m\cos(\phi - \theta)\right\}\cos\theta\right)$$

Here it is seen that the term $\left\{-n - \frac{m^2}{n} + 2m\cos(\phi - \theta)\right\}$ must always be negative whereas the pther terms are always positive. Hence the power represented by the reflected wave of current is negative, which means that it is flowing away from the receiving end of the line.

The conclusions drawn from the foregoing proof, briefly stated, are that the current wave at any point in the line consists of two components flowing in opposite directions and that the voltage at any point in the line consists of two components acting in opposition to each other. Furthermore the power represented by the reflected current wave is lost power.

It might be well at this time to give a summary of the equations and the way in which they pertain to telephone lines.

$$\underset{\cdot}{I} = \frac{\underset{\cdot}{I}_o}{2}\left\{1 + \frac{(r_1 + jx_1)}{(h + jK)}\right\} e^{\alpha l}(\cos\beta l + j\sin\beta l)$$

$$+ \frac{\underset{\cdot}{I}_o}{2}\left\{1 - \frac{(r_1 + jx_1)}{(h + jK)}\right\} e^{-\alpha l}(\cos\beta l - j\sin\beta l) \quad (86)$$

$$E = \frac{I_o}{2}\left\{(r_1 + jx_1) + (h + jK)\right\} e^{\alpha l}(\cos \beta l + j \sin \beta l)$$

$$+ \frac{I_o}{2}\left\{(r_1 + jx_1) - (h + jK)\right\} e^{-\alpha l}(\cos \beta l - j \sin \beta l) \tag{87}$$

$$\alpha = \sqrt{\tfrac{1}{2}(zy + Rq - xb)} \tag{88}$$

$$\beta = \sqrt{\tfrac{1}{2}(zy - Rq + xb)} \tag{89}$$

$$h = \frac{1}{y}\sqrt{\tfrac{1}{2}(zy + Rq + xb)} \tag{90}$$

$$K = \frac{1}{y}\sqrt{\tfrac{1}{2}(zy - Rq - xb)} \tag{91}$$

Now for ideal telephonic transmission the wave sent out from the transmitting end of the line must traverse the line without changing in amplitude or form. That is the wave must not be attenuated nor distorted. If the wave is attenuated then the sound heard at the receiving end has less volume than the sound made at the transmitting end of the line. If the wave is distorted it affects the quality of the sound heard at the receiving end of the line.

In the expression for the attenuation constant,

$$\alpha = \sqrt{\tfrac{1}{2}(zy + Rq - xb)}$$

suppose that q is zero. This is the condition which would exist if the insulation of the line is perfect. Then y equals b and

$$\alpha = \sqrt{\tfrac{1}{2}b(z - x)}$$

Substituting $b = 2\pi fC$

$$z = \sqrt{R^2 + (2\pi fL)^2}$$

$$x = 2\pi fL$$

Then

$$\alpha = \sqrt{\tfrac{1}{2}(2\pi fC)(\sqrt{R^2 + (2\pi fL)^2} - 2\pi fL)}$$

$$\alpha = \sqrt{\pi fC(\sqrt{R^2 + (2\pi fL)^2} - 2\pi fL} \qquad (92)$$

Now the waves transmitted over a telephone line are non-sinnsoidal waves. That is each wave may be expressed as the sum of many sine waves of different frequencies, thus

$$u = U_1 \sin \omega t + U_2 \sin(2\omega t + \alpha) + U_3 \sin(3\omega t + \alpha) \text{ etc.} \qquad (93)$$

where u is the instantaneous value of the wave and U_1, U_2, and U_3 are the maximum values of the component sine waves. α, α, etc., show the phase relations of the harmonics, and ω is equal to $2\pi f$.

From equation No. 92 it is seen that the attenuation constant is dependent on the frequency, waves of high frequency being attenuated more vigorously than waves of low frequency. Suppose a wave of the form expressed by equation No. 93 is transmitted over the line. Each sine component of that wave is attenuated a different amount, the higher harmonics being attenuated more than the lower ones. Thus the wave reaching the receiving end of the line will be of considerably different shape from the wave as it was sent out from the transmitting end of the line and it will also have a smaller amplitude.

Now attenuation in itself is not harmful unless it is excessive because the sending current can be made sufficiently large to carry over long distances, but the unequal attenuation of the various sine components of the wave is harmful in that it distorts the wave to such an amount as to change the quality of the sound transmitted. It is this unequal attenuation of the components of the wave which limits the distance of telephone transmission.

From an inspection of equation No. 92 it is seen that attenuation is dependent on the frequency of the wave transmitted, and on the resistance, capacity and inductance of line. It is also dependent on the leakage but that has been assumed zero. Now the frequency of the transmitted wave depends on the voice frequencies and can not be controlled. If the resistance of the line could be made sufficiently low as to be negligible then attenuation would be null, but this method of overcoming attenuation would be very expensive one and hence can not be employed. There remains, then, two ways in which the attenuation constant can be decreased, either decreasing the capacity of the line or increasing the inductance.

In equation No. 92 expand $\sqrt{R^2 + (2\pi fL)^2}$ by the binomial theorem,

$$\left\{(2\pi fL)^2 + R^2\right\}^{\frac{1}{2}} = 2\pi fL\Big(1 + \frac{1}{2}\frac{R^2}{(2\pi fL)^2} - \frac{1}{8}\frac{R^4}{(2\pi fL)^4} + \frac{1}{16}\frac{R^6}{(2\pi fL)^6}$$

Now if $2\pi fL$ is large in comparison with R the last terms become

negligible. Hence,

$$\left\{(2\pi fL)^2 + R^2\right\}^{\frac{1}{2}} = 2\pi fL(1 + \frac{1}{2}\frac{R^2}{(2\pi fL)^2} \qquad (94)$$

Substituting in equation No. 92

$$\alpha = \left\{\pi fC(2\pi fL + \frac{1}{2}\frac{R^2}{(2\pi fL)} - 2\pi fL\right\}^{\frac{1}{2}}$$

$$\alpha = \left\{\frac{1}{2}\pi fC\frac{R^2}{(2\pi fL)}\right\}^{\frac{1}{2}}$$

$$\alpha = \left\{\frac{CR^2}{4L}\right\}^{\frac{1}{2}}$$

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} \qquad (95)$$

Here it is seen that attenuation is independent of the frequency and hence all components of the wave are attentuated an equal amount. Therefore, as the wave progresses along the line it decreases in amplitude but does not in any way change its form. It must be born in mind that in the derivation of the above formula the conductance was considered zero and the inductance was considered large in comparison with the resistance. Hence, the formula,

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$$

is not exact but does approach very closely the actual conditions in a line of high inductance and good insulation.

It has been shown that by increasing the inductance of a telephone line attenuation can be decreased and that waves of

all frequencies will be attenuated proportionally. It now remains to determine the relative values of the line constants which will give the least attenuation and distortion of the waves. In the formula

$$\alpha = \sqrt{\tfrac{1}{2}(zy + Rq - xb)}$$

Substitute for z and y

$$\alpha = \sqrt{\tfrac{1}{2}(\sqrt{R^2 + x^2}\sqrt{q^2 + b^2} + Rq - xb)}$$

Now as stated above, the minimum value of α obtains at same value of inductance. To find this value of inductance it is necessary to take the first derivative of α with respect to x which is $2\pi fL$, and to solve $\frac{d\alpha}{dx} = 0$

$$\frac{d\alpha}{dx} = \tfrac{1}{2}\left(\left\{\tfrac{1}{2}(\sqrt{R^2 + x^2}\sqrt{q^2 + b^2} + Rq - xb)\right\}^{-\frac{1}{2}}\tfrac{1}{2}\left\{(q^2 + b^2)^{\frac{1}{2}}\right.\right.$$

$$\left.\left.\tfrac{1}{2}(R^2 + x^2)^{-\frac{1}{2}}\,2x + 0 - b\right\}\right)$$

$$= \tfrac{1}{2}\left[\left\{\frac{(q^2 + b^2)^{\frac{1}{2}}x}{(R^2 + x^2)^{\frac{1}{2}}}\right\} - b\right]\frac{1}{\left\{\tfrac{1}{2}(\sqrt{R^2 + x^2}\sqrt{q^2 + b^2} + Rq - xb\right\}^{\frac{1}{2}}}$$

Setting equal to zero

$$(q^2 + b^2)^{\frac{1}{2}}x = (R^2 + x^2)^{\frac{1}{2}}b$$

$$q^2x^2 + b^2x^2 = R^2b^2 + x^2b^2$$

$$q^2x^2 = R^2b^2$$

$$qx = Rb$$

$$\frac{R}{q} = \frac{x}{b}$$

Substituting for x and b, $2\pi fL$ and $2\pi fC$ respectively,

$$\frac{R}{q} = \frac{L}{C}$$

$$L = \frac{CR}{q}.$$

This last equation is the one for a distortionless line. If

$$L = \frac{CR}{q}$$

be substituted in the expression for attenuation,

$$\alpha = \sqrt{Rq} \qquad \text{when } L = \frac{CR}{q}$$

It is seen here that the attenuation is entirely independent of the frequency and hence all waves are attenuated a proportional amount, Therefore, the best wave transmission obtains when $L = \frac{CR}{q}$. It was on this principle that Professor Pupin devised the scheme of inserting artificial inductance at certain intervals along the line thus overcoming to a large extent the distorting effect. The fact that in aerial lines the conductance, q, varies with weather conditions makes it impossible to determine the exact amount of inductance required to give the best results.

After it was found that by increasing the inductance of a telephone line, speech transmission could be improved, it became necessary to find a suitable means of increasing the line inductance. Many methods were experimented with but they all proved to be failures until Professor Pupin brought forth the method of

spacing loading coils at certain pre-determined points. When loading coils, which are simply wire wound inductances with iron cores, were placed at a few points in the line it was found that the transmission of speech was made worse than without the loading coils. This was due to the reflection of the waves between the coils. This is analogous to rope fastened rigidly at one end to a wall. If a wave is made to traverse the rope by giving the free end an up and down movement it will be found that when the wave reaches the wall it will be reflected back again and interfere with the succeeding waves. However, it is possible to so time the incoming wave that the reflected wave will have the same phase position as the incoming wave and hence will not cause any interference. On this principle Professor Pupin developed a formula for the correct spacing of loading coils.

Let d = distance between two adjacent loading coils.
λ = wave length.
ϕ = angular distance between loading coils.
$2\pi = 360^\circ$

Then

$$\frac{\phi}{2\pi} = \frac{d}{\lambda}$$

$$d - \lambda \frac{\phi}{2\pi}$$

It was shown in equation No. 81 that

$$\lambda = \frac{2\pi}{\beta}$$

Therefore,

$$d = \frac{\phi}{\beta}$$

Now in an unloaded line the inductance is uniformly distributed over the length of the line but in a loaded line this is not true because of the inductance coils being inserted at points along the line. If the artificial inductance could be inserted in the line in such a manner as to be uniformly distributed over the length of the line the wave reflection between coils would be entirely overcome and the loaded line would be equivalent to an unloaded line of the same inductance. But as stated above it is impossible to insert artificial inductance uniformly hence a loaded conductor is not equivalent electrically to an unloaded line of the same inductance. Professor Pupin found that the degree of equivalence between a loaded conductor and an unloaded conductor of the same inductance is as the ratio sin $\frac{\phi}{2}$ to $\frac{\phi}{2}$. Thus the best transmission of speech is obtained when,

$$\sin\frac{\phi}{2} = \frac{\phi}{2}$$

or substituting for $\phi = \frac{2\pi d}{\lambda}$

$$\sin\frac{\pi d}{\lambda} = \frac{\pi d}{\lambda}$$

But this condition would necessitate having zero distance between coils or in other words a uniformly distributed inductance. However in practice the distance between the coils is usually made about one-tenth the distance of a wave length. The degree of equivalence in this case is

$$\frac{\sin(0.1\pi)}{.1\pi} = \frac{.309}{.314} = .983$$

This means that a loaded conductor is electrically equivalent to

an unloaded conductor of the same inductance to within 1.7 %.

Now it has been shown, theoretically, how the transmission of waves over a telephone line can be improved by increasing the inductance of the conductors. It has also been shown that the reflected wave means a flow of power back toward the generating end of the line and a greater loss of power in the line for a given power delivered than would occur if the reflected wave were eliminated. Therefore, if a means could be employed which would eliminate the reflected wave or at least reduce it to a negligible amount, the transmission of speech would be greatly improved. The most feasible way to bring about the elimination of the feflected wave is to effect a change in the terminal constants of the line.

In equation No. 86 it is seen that the constant factor of the incoming wave is

$$\frac{I_o}{2}\left\{1 + \frac{(r_1 + jx_1)}{(h + jK)}\right\}$$

and for the reflected wave

$$\frac{I_o}{2}\left\{1 - \frac{(r_1 + jx_1)}{(h + jK)}\right\}$$

CHAPTER III.

The Determination of the Terminal Constants.

In the preceding chapter it has been shown that the magnitude of the reflected wave is determined by the relative values of the line constants and of the terminal constants. While the line constants, with exception of the leakage conductance, can be determined by calculation, the terminal constants, that is the resistance and inductance of the receiving station, can only be determined by measurement. More over, since the hysteresis and eddy current losses in the iron and the magnitude of the vibration of the receiver diaphragm all vary with the frequency, the receiver circuit cannot be regarded as a circuit of fixed resistance and inductance. The equivalent resistance and inductance of the receiving circuit must be measured at frequencies of the same order as the important voice frequencies. The most important voice frequencies have been found to be between 600 and 2000 cycles per second, hence it is at these frequencies that the equivalent resistance and inductance must be found.

The first method used for securing frequencies of the same order as the important speach frequencies was to separate the non-sinusoidal wave of an alternating current generator into its various sine components or harmonics. In order to separate the various harmonics from the fundamental, the generator selected, a three phase star wound machine, was connected to transformers, as shown in Fig. 3. In this figure three single phase transformers, having their primary windings connected in star, are shown

Now h and K are line constants but r_1 and x_1 are constants of the terminal apparatus. In the expression for the constant term of the reflected wave it can be seen that if $\frac{r_1 + jx_1}{h + jK}$ could be made equal to unity the reflected wave would have a zero value. However, a difficulty presents itself in that the term $\frac{r_1 + jx_1}{h + jK}$ can not be made equal to unity at all frequencies. r_1, x_1, h and K are all dependent on the frequency. Since in telephone transmission the frequency of the waves is constantly changing it is practically impossible to eliminate the reflected wave by making $\frac{r_1 + jx_1}{h + jK}$ equal to unity. However, there is some certain value which would give a reflected wave of the least value.

Consider a line whose constants are so proportioned as to give the least attenuation of the waves. That is

$$L = \frac{CR}{q}$$

or

$$x = \frac{bR}{q}$$

Substituting these values in equations Nos. 64 and 65 for h and K and solving

$$h = \sqrt{\frac{R}{q}}$$

$$K = 0$$

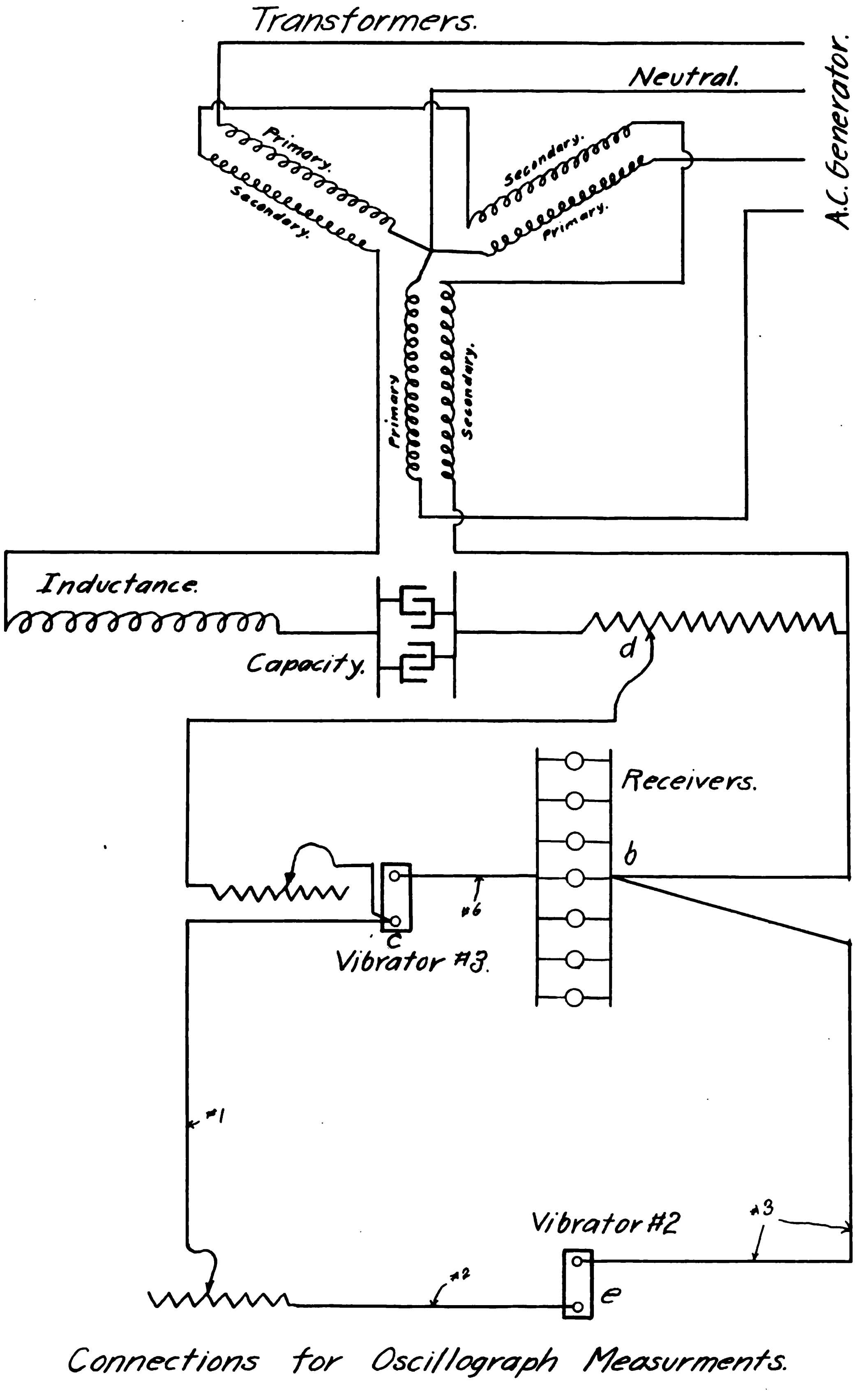
Here it is seen that both h and K are independent of the frequency in a distortionless line. Substituting in the expression for the constant term of the reflected wave

$$\frac{\underset{\bullet}{I}_o}{2}\left\{1 - \frac{r_1 + jx_1}{h + jK}\right\} = \frac{\underset{\bullet}{I}_o}{2}\left\{1 - \frac{r_1 + jx_1}{\sqrt{\frac{R}{q}}}\right\}$$

Now if r_1, the resistance of the terminal apparatus, and x_1, the reactance, were independent of the frequency it would be a simple matter to eliminate the reflected wave, but such is not the case. However, it can be seen that r_1 and x_1 can be given values which will produce the minimum reflected wave.

To summarize briefly, it has been shown that long-distance telephony has been hampered by attenuation, distortion and reflection of the waves. Loading coils are serving their purpose in overcoming, to a large extent, attenuation and distortion of the waves. Furthermore their are several different types of terminal apparatus which reduces the reflected wave to a minimum but as yet no apparatus has been discovered which will entirely eliminate the reflected wave.

Fig. 3.



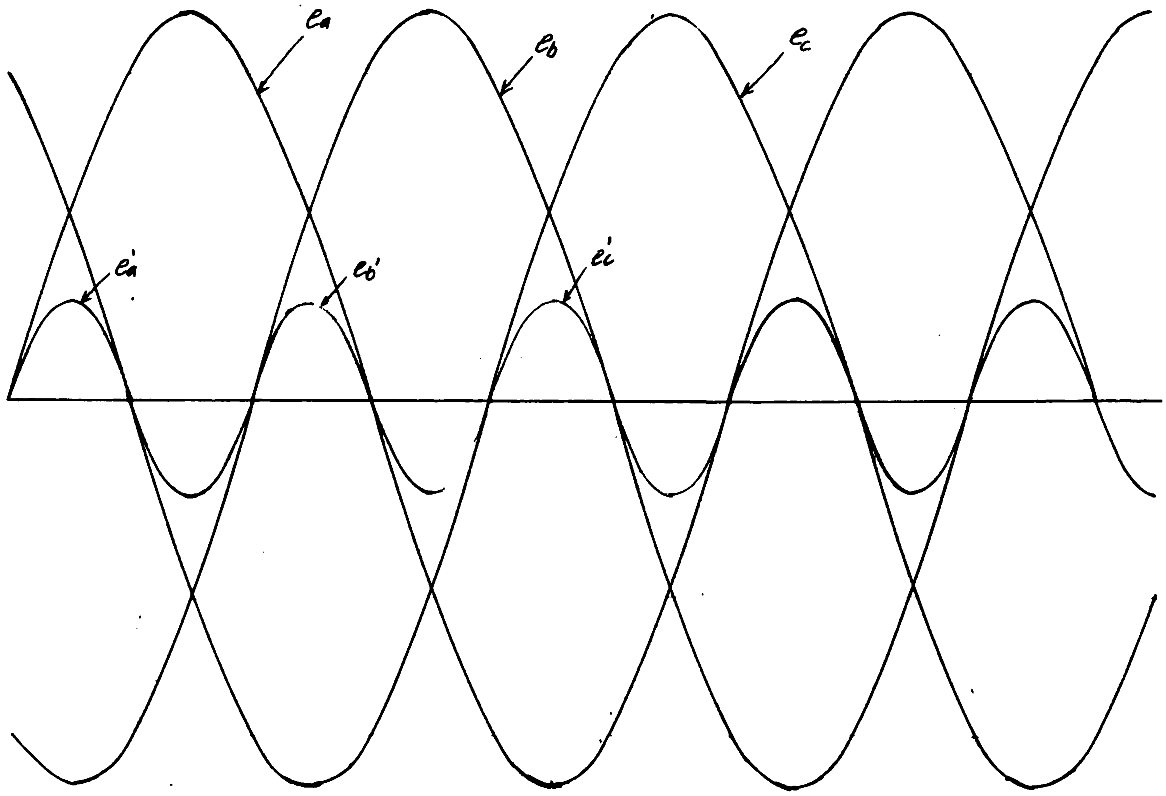
Connections for Oscillograph Measurments.

connected to the three terminals of the star wound generator. The neutral point of the star connected transformers is also connected to the neutral of the generator. The secondary coils of the transformers are shown connected in delta with the delta connection closed through the circuit whose resistance and inductance is to be measured. An examination of the voltage relations in the open delta circuit will show that the fundamental has been eliminated between the ends of the open delta, since the sum of the voltages in a three phase delta connection is zero. This is shown clearly by Fig. 4. In Fig. 4, e_a is the voltage wave in phase one, e_b the voltage wave in phase two, displaced from e_a by 120°, e_c shows the voltage wave in phase three, displaced from e_a by 240°. At any instant, then, it is seen that the sum of e_b and e_c is equal and opposite to e_a, hence the voltage across the delta is zero as far as the fundamental is concerned. Now to show that there are higher harmonics in the delta. In Fig. 4, e_a, e_b, e_c, are the fundamental voltages as explained above, e_a' is the third harmonic corresponding to e_a, and e_b' and e_c' are the third harmonics corresponding to e_b and e_c respectively. It is readily seen from this figure that the three waves representing the third harmonics, e_a', e_b', e_c' of the three fundamentals e_a, e_b, e_c, are in phase and that the total voltage is the sum of the three. This can also be shown as follows

Let $e_a' \sin 3\omega t$ = Voltage of third harmonic in phase 1 at any instant

$e_a' \sin 3(\omega t + 120)$ = Voltage of third harmonic in phase 2 at any instant.

Fig. 4.



Delta Voltages.

and

$e_c' \sin 3(\omega t + 240)$ = Voltage of third harmonic in phase 3 at any instant.

Adding gives,

$$e_a' \sin 3\omega t + e_b' \sin 3(\omega t + 120) + e_c' \sin 3(\omega t + 240)$$

or

$$e_a' \sin 3\omega t + e_b' \sin 3(\omega t + 360) + e_c' \sin 3(\omega t + 720)$$

which is

$$(e_a' + e_b' + e_c') \sin 3\omega t$$

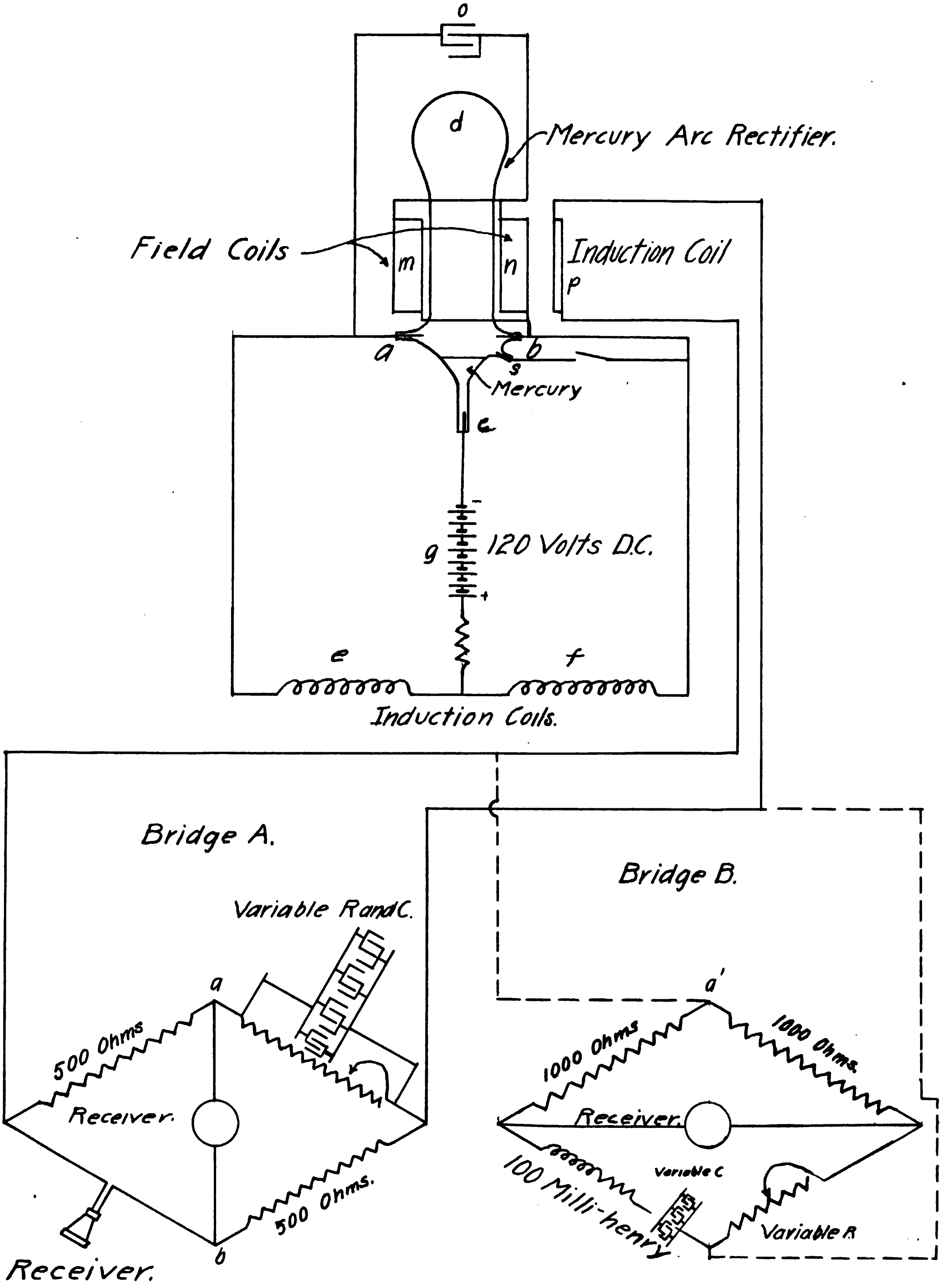
Thus it is seen that the third harmonic is present and has a value equal to the sum of the values in all three phases. Similarly it can be shown that the ninth and all other harmonics which are multiples of three are present, if they are present in the non-sinusoidal wave. The connection as shown in Fig. 3 then give a means of separating the fundamental from the harmonics of higher frequency.

In the preceding paragraph a method has been given for separating the fundamental from the higher harmonics. An examination of the wave in the open delta connection showed that only the third and ninth harmonics were of sufficient magnitude to be measurable. Hence this method would give frequencies no higher than 540 cycles per second, the fundamental being 60 cycles. However, it was thought desirable to make measurements with the frequencies obtainable by this method, so a means of separating the third harmonic from the ninth was devised. The method used was to make the circuit resonant for the desired harmonic. If the circuit is resonant for one harmonic the impedance to the other will be so high that the current which will flow will have a very

small value in comparison to the one for which the circuit is resonant. If, then, the condition $2\pi fL = \frac{1}{2\pi fC}$ is fulfilled for one harmonic, the value of the current in the circuit will be large for this harmonic in comparison with the other. Thus a means of separating the third and ninth harmonic is obtained.

The highest frequency to be obtained from the non-sinnodial wave of the generator which could be used for the purpose of measurements was 540 cycles. It is necessary, however, to secure higher frequencies. These higher frequencies were obtained from the Vreeland Oscillator connected as shown in Fig. 5. Refering to the figure D represents a mercury arc rectifier rated at 20 amperes, 110 volts. a and b are the two positive electrodes and e the negative electrode and s the starting electrode, connected by mercury as shown. E and F are two similar inductance coils and G is a 120 volt battery. M and N are two similar coils of low resistance, O is a variable capacity and P is an inductance coil of a few turns of wire. The connections are as follows. From the positive side of the battery a lead is carried through a resistance which limits the flow of current. From this resistance leads are carried to the two similar inductance coils E and F. From the inductance coil E a lead is run to electrode a and from coil F to electrode b. The circuit from the battery is completed through electrode e to the negative side of the battery. From the starting electrode s a lead is run through a single pole switch and resistance to the electrode b. This completes the connections for what will be called the D.C. circuit. The cir-

Fig. 5.



Connections for Oscillator and Bridges.

cuit.containing the coils M and N will be called the A.C. circuit and is connected as follows. Electrode b is connected to the coils M and N, these coils being in parallel in such a manner that the flux which any current in them produces is in the same direction in both. The other ends of the coils M and N are connected to the condenser O and O is connected to electrode a. The connections of the coil P will be explained later. Now the action of the oscillator is as follows. To start the arc, the tube is tipped until the mercury from the chamber around electrode c comes into contact with electrode s, the starting electrode, which has been connected to electrode b by closing the switch. When the tube is tipped back again the mercury draws away from electrode s and an arc forms between the electrode s and the mercury. This arc continues to pass between the electrode s and the mercury until the starting switch is opened, when it jumps to the electrodes a and b. Since the coils E and F have the same resistance and the distance from a and b to the mercury are the same, the current from the battery will be equally divided in the two branches, hence the electrodes a and b will take the same current. If, however, the arc should move towards either of the electrodes a or b, a small distance the paths from the electrodes to the mercury will no longer be the same and hence the current taken by the electrodes will be different. The coils E and F tend to keep the current the same and hence the excess of current taken by one electrode must go through what has been called the A.C. circuit, that is the circuit containing the field coils M and N and capacity O. The flow of current through the field coils sets

up a flux through the coils. This flux passes through the Rectifier tube and causes the arc stream to be blown still further toward the electrode from which the greatest current comes. This causes still more current to flow through the A.C. circuit. When the condensers in the A.C. circuit become fully charged no more current will flow through them and the current must again be equally divided between the two electrodes. When the current is equally divided between the electrodes a and b there is no difference of potential between the electrodes and have nothing to prevent the condensers from discharging. When the condensers discharge an excess of current flows through the electrode that had the least before. The process just described above will then be repeated for this electrode. At first one and then the other electrode carrying an excess of current. This process will be repeated as many times per second as the constants of the circuit will permit. Thus by proper proportioning of the inductance and capacity the frequency of the current in the A.C. circuit can be made to vary over a wide range. With the apparatus used for the oscillater described above, the limits between which the frequencies could be varied was found to be 500 and 1800 cycles per second.

The preceding paragraphs have given the methods devised for securing currents of frequencies which are of the same order as the important speach frequencies. It now remains to show how these high frequency currents were used for making the desired measurements. For frequencies as high as 600 cycles per second the method used was to take oscillograms of the current through

the apparatus and the voltage across the apparatus where resistance and inductance was to be measured. At frequencies higher than 600 the vibrators of the oscillograph reflected such a narrow beam of light and the length of one wave length was so small that the oscillograms taken were not satisfactory for making measurements. Hence oscillograms were taken only for the frequencies below 600 cycles per second. For all frequencies above 600 the alternating current bridge method was used.

In making measurements on the resistance and inductance of the telephone apparatus by use of oscillograms a General Electric Oscillograph connected as shown in Fig. 3 was used. In this figure the open ends of the delta are shown connected to the circuit made resonant for the desired harmonic. The resistance in series with the resonant circuit is shown shunted by a circuit a, b, c, d, which contains the apparatus to be measured, and a vibrator of the oscillograph. The vibrator in this circuit then shows the current flowing through the apparatus to be measured. The circuit a-b-c-d is shown shunted by a second circuit b-e-c containing vibrator No.2 and a variable resistance. This second vibrator then shows the voltage across the circuit b-c. If then an oscillogram be taken of the current and voltage as shown by the vibrators, it will show the phase relations of the current and voltage. Then if the value of the current and voltage be known the value of the equivalent resistance and inductance can be found In order to know the value of the current and voltage, the vibrators must be calibrated so that a given deflection represents a

certain known current. The calibration of the vibrators is obtained by means of the connections shown in Fig. 6.

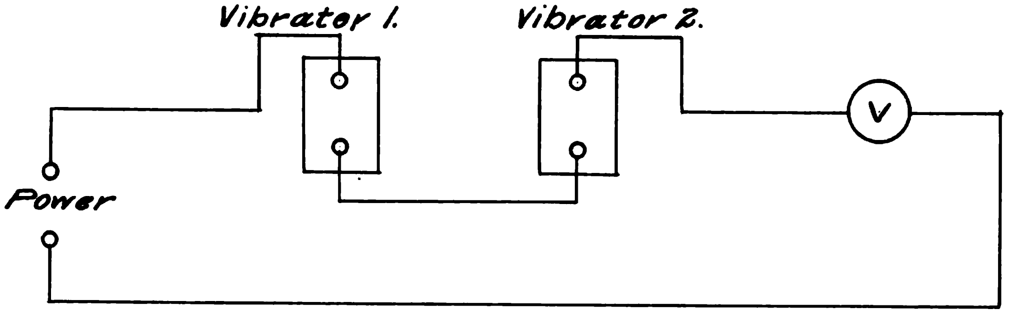


Fig. 6.

The vibrator and a voltmeter of known resistance are connected in series and then connected to the power supply from which the current for the measurements is to be taken. The current through the vibrator and voltmeter is given by the reading of the voltmeter divided by the resistance of the circuit. Then if an oscillogram be taken of the deflection produced by the known current, the current which will cause one inch deflection can be determined and hence the current represented by any other deflection can readily be determined. If now oscillograms be taken with connections as shown in Fig. 3, the deflection of Vibrator No.3 will give the current in circuit b-c and the deflection of vibrator No. 2 will give the current in circuit b-c-e. The current through b-c-e- multiplied by the resistance of leads 1,2,and 3, and the resistance R and the resistance of vibrator No. 2 will give the voltage across circuit b-c. Hence the oscillograms give the value of current and voltage and if there is any phase displacement between the current and voltage it will also be shown. Hende the values of the equivilent resitance and inductance can be found.

When the frequency was higher than 600 cycles, it was then necessary to resort to the A.C. bridge. Fig. 5 shows the connections used with the bridge measurements. Bridge A in this figure is an A.C. bridge having two equal resistances on opposite sides of the bridge, one connected to each power lead. The other sides contain the unknown inductance and resistance, as a receiver, and a variable capacity bridged across a variable resistance. A telephone receiver is connected across a and b of the bridge. The resistance of the receiver was found by balancing the variable resistance against the receiver resistance. The inductance was found by balancing the variable capacity against the inductance. When both were balanced the current through the balancing receiver was zero. This method is independent of the frequency and hence should be very accurate. To prove that this method is independent of the frequency,

Let, L = unknown inductance-henrys

R = unknown resitance

R_1 - known resistance

C = known capacity-farads.

Then
$$\frac{500}{R + j2\pi fL} = \frac{1}{\frac{\frac{1}{R_1} + j2\pi fC}{500}}$$

Clearing of fractions,
$$250000\left(\frac{1}{R_1} + j2\pi fC\right) = R + j2\pi fL$$

Equating real and imaginary terms,
$$250000 \times \frac{1}{R_1} = R$$

$$250000 \times 2\pi fC = 2\pi fL$$

Therefore,

$$L = 250000C = 0.25C \text{ where } C \text{ is in m.f.}$$

Thus all frequencies can be balanced out with this bridge. This method was used in making measurements on the receiver, the coil, and the complete telephone set. It did not, however, give any idea of the frequency. Bridge B, Fig. 5, was used to determine frequencies. Two resistances of 1000 ohms each were placed in the upper branch of the bridge. In the lower branch were placed a standard inductance of 100 milli-henrys and a variable resistance and capacity. A telephone receiver was connected between the upper and lower branches to detect the balance. The equations for the bridge are as follows:

Let L = standard inductance in henrys

C = variable capacity in farads

R = resistance of standard inductance

R_1 = variable resistance

w = frequency

Then,

$$\frac{1000}{R + jwL - 1/wC)} = \frac{1000}{R_1}$$

Clearing of fractions

$$1000(R_1) = 1000(R + jwC - j\frac{1}{wC})$$

Equating real terms and imaginary terms,

$$R_1 = R$$

$$\frac{1}{wC} = wL$$

Therefore,

$$w^2CL = 1$$

$$w = \frac{1}{\sqrt{CL}} = 2\pi f$$

$$f = \frac{1}{2\pi\sqrt{CL}}$$

Thus by balancing the bridge and noting the value of the capacity, the frequency was obtained. The frequencies were determined for certain values of capacity in the A.C. circuit with a certain current from the battery. The current was then held constant and the value of the capacity changed to secure different frequencies. After making determinations of the frequencies, with various capacities in the A.C. circuit, the same frequencies could be obtained when desired by placing the required capacity in the circuit. Hence, it was necessary to make but one determination of the frequency for each value of capacity inserted.

Methods of Computing Results.

The calculations fo the resistance and inductance was made from the oscillograph records in the following manner:- A calibration curve was taken with a known current flowing through the vibrators of the oscillograph. This known current, the voltage shown on the voltmeter divided by the total resistance of the circuit, produced a given deflection of the beam of light. Such a calibration curve is shown diagrammatically in Fig. 7, The total deflection I on this curve then represents two times the maximum current in the vibrator. The maximum current, however, is the current calculated from the voltmeter reading times the square root of two, $I_m = I_{eff} \times \sqrt{2}$. Each inch of deflection then

represents a certain current which is given by the equation

$$1 \text{ inch} = \frac{\frac{E}{R} \times \sqrt{2}}{\frac{1}{2} \text{ double deflection}} \text{ amperes.}$$

The double deflection is the distance from the peak of the wave above zero to the peak below. Then to find the current represented by a wave of some other height, as in curve 2, Fig. 7, it is only necessary to measure the distance from the top of one loop to the bottom of the next and multiply by the value found for one inch. This will give two times the maximum value of the current. Then to find the true effective value it is necessary to divide by two times the square root of two. The equation for finding the effective value of a current from a wave having a double deflection as I_1, curve 2, Fig. 7, can be expressed in this way.

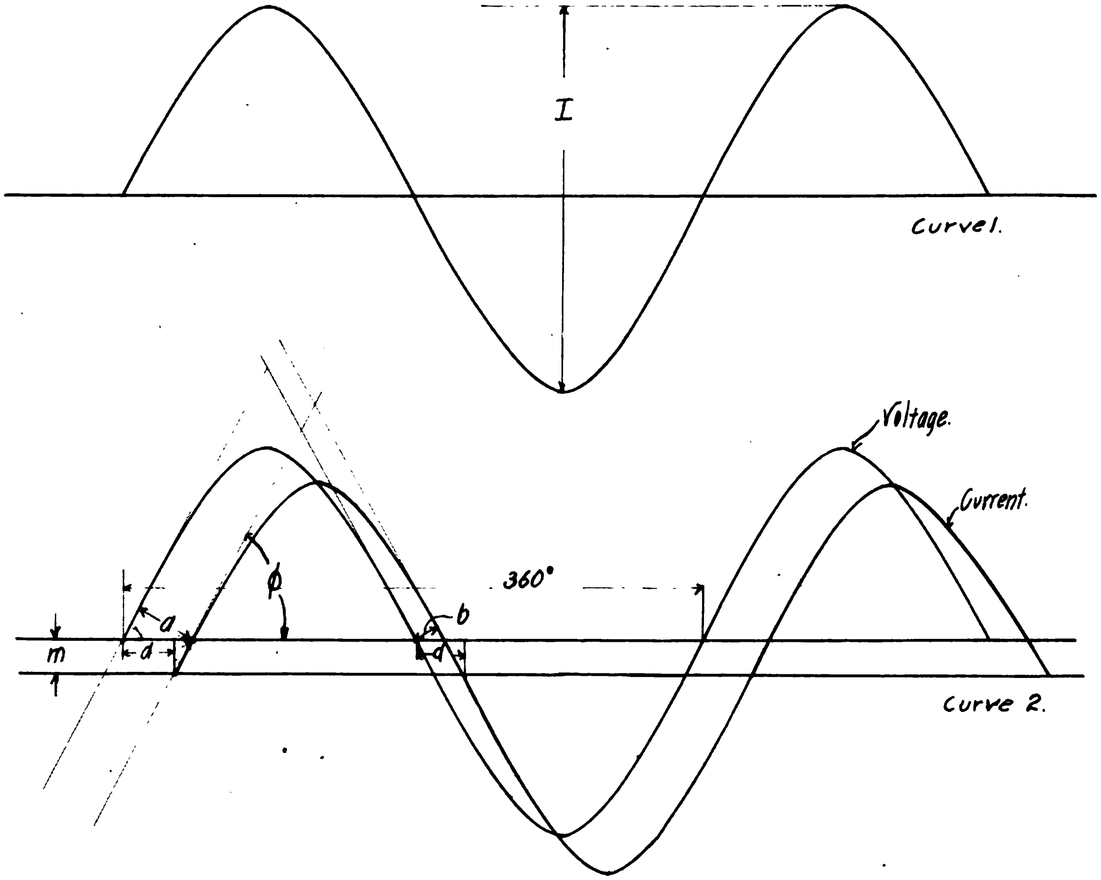
$$\text{Effective } I = \frac{E/R \times \sqrt{2}}{\frac{1}{2} \text{ double deflection}} \; \frac{I_1}{2\sqrt{2}}$$

or

$$I = \frac{E/R \times I_1}{\text{double deflection}}$$

When the vibrators have been calibrated in this manner, the current represented by any deflection can be found. The current flowing in the two vibrators is then determined for the curves taken with the apparatus to be measured in the circuit. The value of the current in the current vibrator gives the value of the current in the receivers while that in the voltage vibrator gives a means of measuring the voltage. This voltage is then the current, in the voltage vibrator, multiplied by the total resistance

Fig. 7.



in the circuit. In Fig. 3 this resistance is, leads 1, 2, 3, the resistance box, and vibrator No. 2. With this voltage determined, and the current through the apparatus being known, the impedance of the circuit is $z = \frac{E}{I}$. Now if the phase angle were known, the resistance and reactance could be easily found for

$$R = Z \cos \theta \quad \text{and} \quad x = Z \sin \theta \quad \text{where,}$$

θ is the angle of phase difference. There are two methods of determining this phase angle. Curve 2 Fig. 7, shows two waves, not on the same zero line, whose maximum deflections are nearly equal and which are out of phase by some small angle. If corresponding points on the curves be projected to the same zero line, the distance between the points will be a measure of the phase difference. That is

$$\text{Phase difference} = 360^\circ \frac{\text{Displacement on same zero line}}{\text{Length of 1 cycle.}}$$

This method is, however, not accurate when the wave length is small, hence another method was devised. This consisted of drawing two parallel lines through corresponding zero points of the curves. These lines will be tangent to the curve for some distance, for the curves are nearly straight, close to the axis. Then the phase angle can be found in this way. Measure the perpendicular distance between the two sets of parallel lines, as distances a and b, Fig. 7, and the angle the lines make with the axis. Let d, Fig. 7, be the phase displacement. Then

$$d(\text{on the left}) = \frac{a}{\sin \phi} - \frac{m}{\tan \phi}$$

where ϕ is the angle between the parallel lines and the zero line and m is the distance between zero lines. Also,

$$d(\text{on the right}) = \frac{b}{\sin\phi} + \frac{m}{\tan\phi}$$

Adding the values of d,

$$2d = \frac{a}{\sin\phi} + \frac{b}{\sin\phi}$$

Therefore,

$$d = \frac{a + b}{2\sin\phi}$$

The phase difference, then is the same as before.

$$\text{Phase angle } \theta = \frac{d}{\text{Length of 1 cycle}} \times 360^\circ$$

If, though, the lower wave was so low that it always fell below the upper, the distance d on the left would be d

$$d = -\frac{b}{\sin\phi} + m\cos\phi$$

and the resultant d would be

$$d = \frac{a - b}{2\sin\phi}$$

When this phase angle has been found the resistance and reactance of the apparatus being measured can be found from the equations as given above.

$$R = z\cos\theta \qquad \text{and} \qquad x = z\sin\theta$$

This resistance, however, gives the resistance of the current vibrator as well as of the resistance, hence the resistance of the current vibrator and lead six must be subtracted. The result-

ant resistance is then the resistance of the apparatus. In all these tests there were twenty pieces of apparatus, in parallel, in the circuit. Shown as receivers in Fig. 3. The resistance of each would then be twenty times that determined above. The reactance obtained above is next used to obtain the inductance from the formula,

$$x = 2\pi fL$$

where L is the inductance and f the frequency used. This L must also be multiplied by 20 to obtain the inductance of one piece of apparatus. Thus by use of the oscillograph the resistance and inductance of the apparatus was determined with fair accuracy.

The reason for the use of twenty pieces of apparatus was that the value of the current taken by one, under average telephonic conditions, would be too small to cause an appreciable deflection of the vibrators. The current taken by twenty could, however, be easily measured. When measurements were made upon the complete telephone set, there were not sufficient transmitters on hand. To compensate for the transmitters a constant resistance equal to the average resistance of a number of transmitters was inserted in the circuit. This would cause but slight errors because the variations in resistance of the transmitters would average to approximately the value used.

The method used in making the bridge measurements differed greatly from those used with the oscillograph. As stated before satisfactory oscillograms could not be obtained at frequencies higher than 600 cycles per second. In the bridge method the

high frequency current was obtained from the induction coil shown in Fig. 5. The apparatus to be measured was placed as shown by the receiver in Fig. 5. When there was no sound in the balancing receiver the bridge was balanced and the equations

$$R = \frac{250000}{R_1}$$

and

$$L = 250000\ C$$

gave the values of the inductance and resistance of the apparatus being measured. The receiver, the coil, and the telephone set, as used in practice, were measured in this manner.

The frequency was determined by use of bridge B, Fig. 5 The capacity used in the A.C. circuit of the oscillator was grouped in groups of about 3 micro-farads each. The bridge was set;up once and readings taken ont the frequency with various groups of capacity in the circuit, the current being constant at all times. Having once determined the frequency resulting with a certain group capacity in the circuit, the same frequency could be obtained at any time by simply using the same value of capacity. Thus it was necessary to make but one determination of frequency, which was obtained from the equation of the bridge derived previously.

$$f = \frac{1}{2\pi\sqrt{CL}}$$

The range in frequency was from 600 cycles to 1800 cycles. Intermediate steps were used, the steps being about 200 cycles.

CHAPTER IV.

Sample Calculations, Tabulated Results and Curves.

Oscillograph Measurements.

To illustrate the methods used in computing the resistance and inductance of the apparatus consider Fig. 12, which is an oscillograph record of the current and voltage of twenty receivers connected in parallel. It was first necessary to calibrate the current and voltage vibrations. The calibration oscillogram is shown in Fig. 8.

By measurement the double deflection of the current curve Fig. 8 is 0.525".

Voltmeter reading - 31 volts.

Voltmeter resistance - 998 ohms.

Frequency - 175.5

$$1 \text{ inch} = \frac{\frac{E}{R}\sqrt{2}}{\frac{1}{2} \text{ double deflection}} \text{ peak amperes.}$$

$$1 \text{ inch} = \frac{31}{998} \; \frac{\sqrt{2}}{\frac{0.525}{2}} = 0.167 \text{ peak amperes.}$$

Similarly it is found that one inch deflection on the voltage curve is equal to 0.175 peak amperes.

From the oscillogram of the receivers, Fig. 12 the following measurements were made.

Double deflection of current curve = 1.78"

Double deflection of voltage curve = 1.66"

Length of one cycle or 360^0 = 0.95"

Distance - a (See Fig. 7) = 0.13"

Distance - b (See Fig. 7) = 0.03"

Angle - θ (See Fig. 7) = $81^0 15'$

Frequency = 3 x 58.5 = 175.5 cycles per second.

To calculate the angle by which the current lags behind the voltage.

$$\text{Angle of lag} = \frac{a - b}{2\sin\theta} \times \frac{360}{\text{length of } 360^0}$$

$$= \frac{.13 - .03}{2 \times .9884} \times \frac{360}{.95}$$

$$= 19^0\ 12'$$

$$\text{Value of current} = \frac{1.78 \times .167}{2\sqrt{2}} = .105 \text{ amperes.}$$

Value of current through voltage vibrator equals

$$\frac{1.66 \times .175}{2\sqrt{2}} = .103 \text{ amperes.}$$

Then the voltage across the receivers, lead No. 3 and current vibrators is equal to the product of .103 and the resistance of the leads 1, 2, and 3 plus the variable resistance. (See Fig. 3)

$$\text{Voltage} = .103 \times .905$$

$$= .932 \text{ volts.}$$

The impedance of the twenty receivers connected in parallel, lead 6 and the current vibrator is

$$z = \frac{E}{I} = \frac{.932}{.105}$$

$$z = 8.85 \text{ ohms}$$

$$R = z \cos \theta$$

$$X = z \sin \theta$$

$$R = 8.85 \cos 19^{\circ}\ 12' = 8.36 \text{ ohms}$$

In order to obtain the resistance of the receivers alone the resistance of lead No. 6 and of the current vibrator must be subtracted.

Thus, $8.36 - .73 = 7.63$ the resistance of twenty receivers in parallel;

$7.63 \times 20 = 152.6$ ohms the resistance of one receiver.

$$X = x \sin \theta = 8.85 \times \sin 19^{\circ}\ 12'$$

$$= 2.91 \text{ ohms.}$$

$$X = 2\pi fL = 2.91$$

$$L_{20} = \frac{2.91 \times 1000}{2\pi 175.5} = 2.64 \text{ mili henrys}$$

The inductance of one receiver is then

$$20 \times 2.64 = 52.8 \text{ mili henrys.}$$

In a similar manner the resistance and inductance of the induction coils and telephone sets were determined for frequencies up to 550 cycles.

Frequency - 177.5

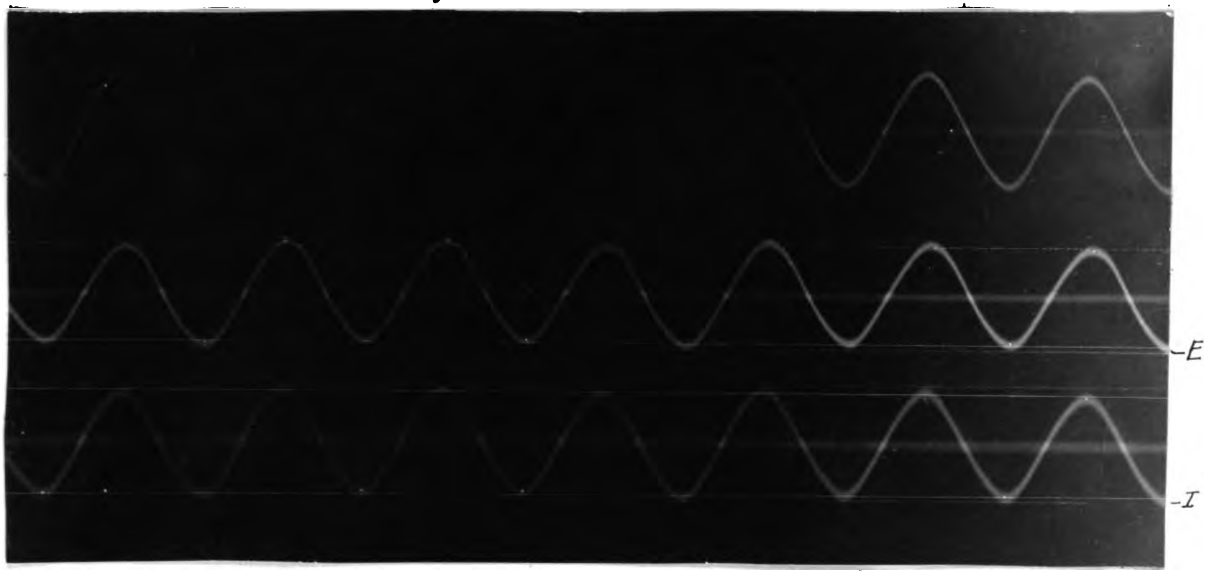


Fig. 8

Frequency - 62

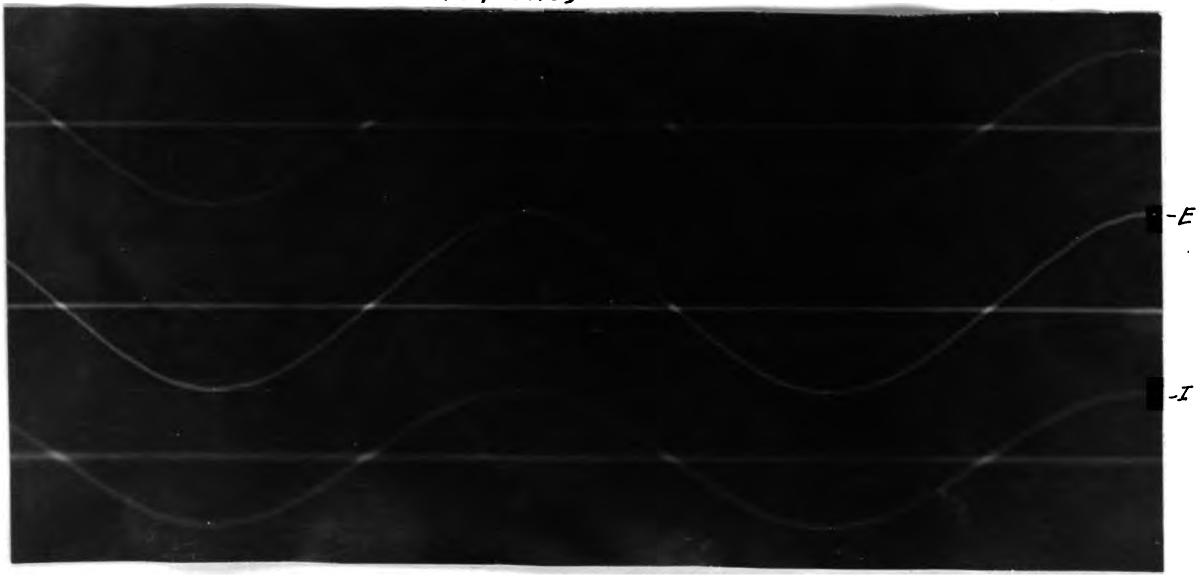
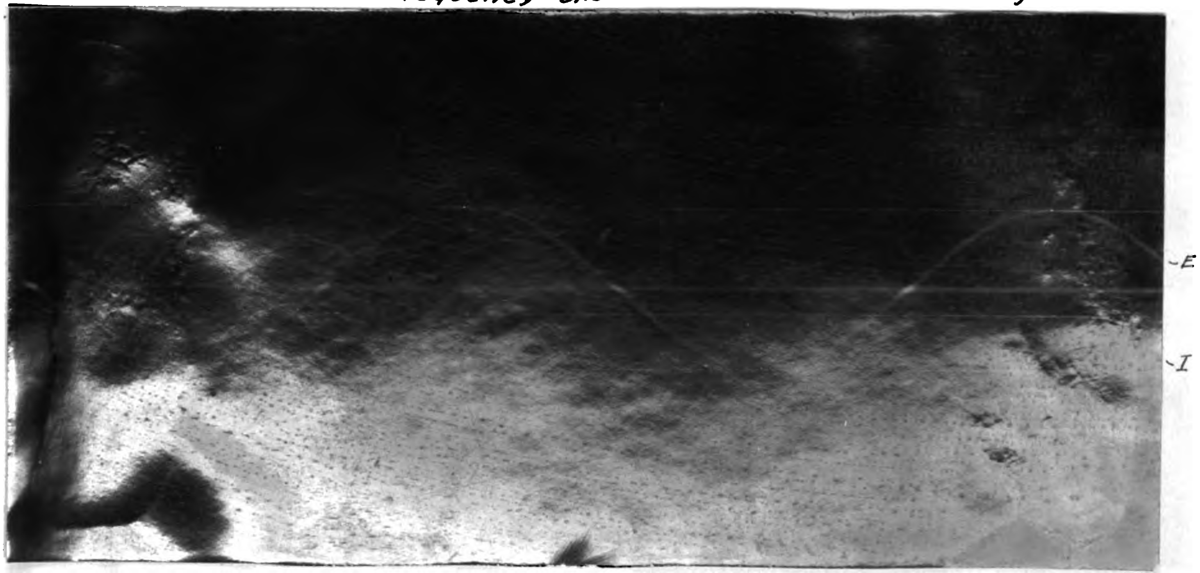


Fig. 9

Frequency - 61.5



Calibration Oscillograms

Fig 10

Frequency - 61.5

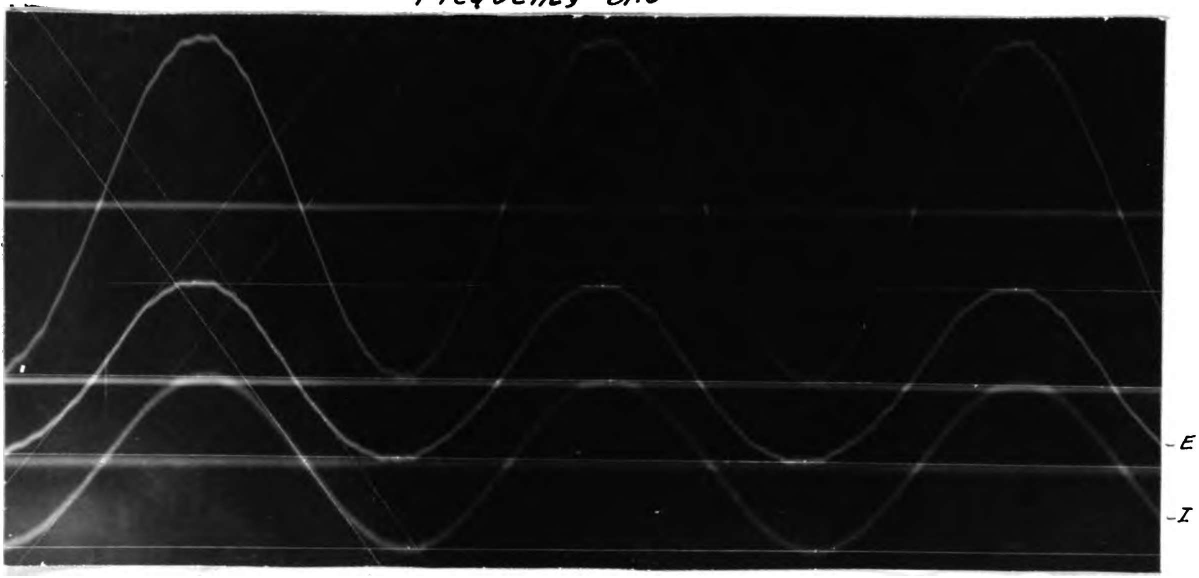


Fig. 11

Frequency - 175.5

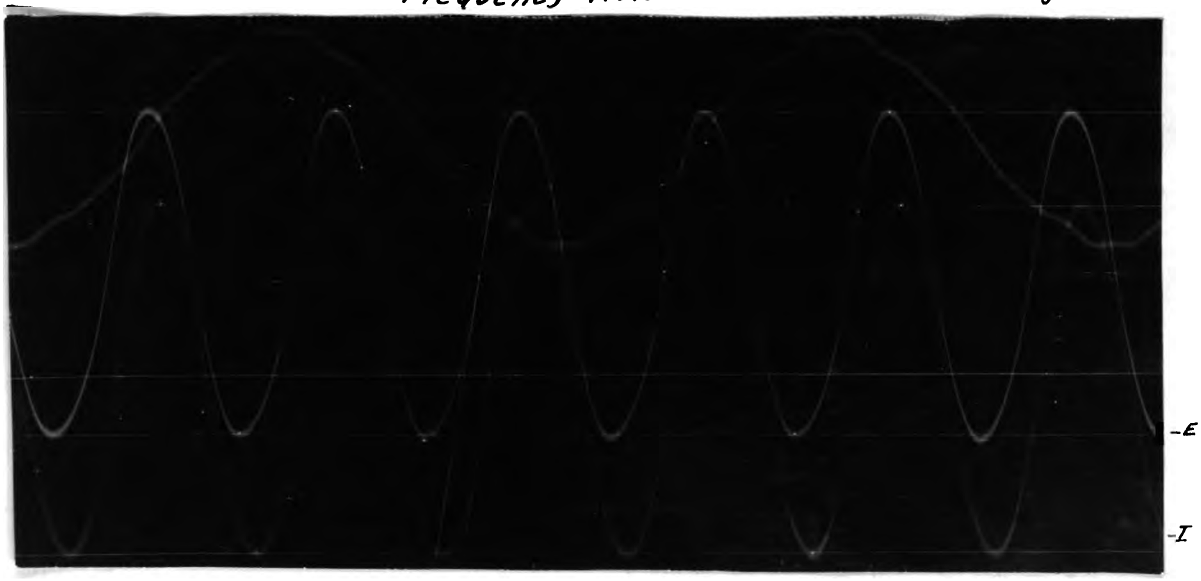
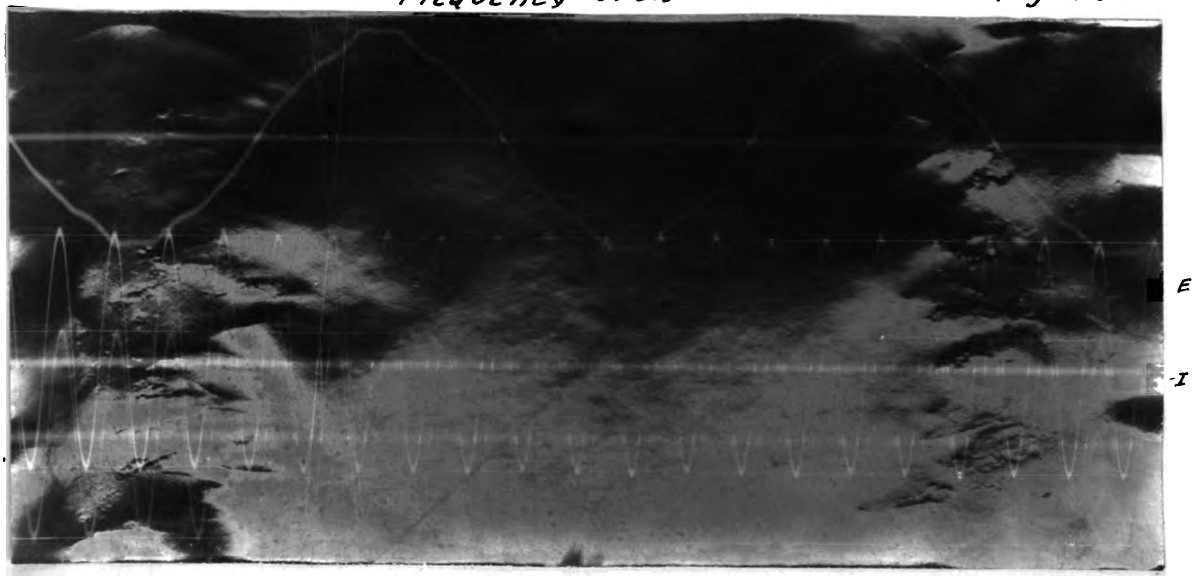


Fig. 12

Frequency - 526.5



Receiver Oscillograms

Fig. 13

Calculated Results from Oscillograph Records.

Calibration Oscillograms.

Number	Figure number	Double deflection current vibrator, In.	Double deflection voltage vibrator, In.	Volt meter reading Volts.	Volt meter resistance. Ohms.	One in. equivalent current vibrator, Amps.	One in. equivalent voltage vibrator, Amps.
1	8	.525	.500	31	998	.1671	.1753
2	9	.710	.935	110.5	2506	.1754	.1332
3	10	.695	.820	110	2506	.1784	.1510
4	--	.880	1.02	110	1996.2	.1770	.1528

Resistance and Inductance Determinations of
Western Electric Receiver. Catalogue No.143 A.W.

Oscillogram Measurements.

Number of Figure	Calibration curve, Number.	Double deflection current vibrator, In.	Double deflection voltage vibrator, In.	Length of 360° Inches	Angle θ Degrees	a Inches	b Inches	Frequency cycles per sec.
11	1	.87	.905	2.1	$52^\circ 40'$	.315	.215	61.5
--	1	1.78	1.64	.8	$82^\circ 20'$	.105	.018	175.5
12	1	1.78	1.66	.95	$81^\circ 15'$	.130	.030	175.5
--	1	1.08	1.245	.45	$83^\circ 20'$	.085	.040	526.5
13	1	1.06	1.19	.29	$85^\circ 40'$	.055	.035	526.5

Calculated Data.

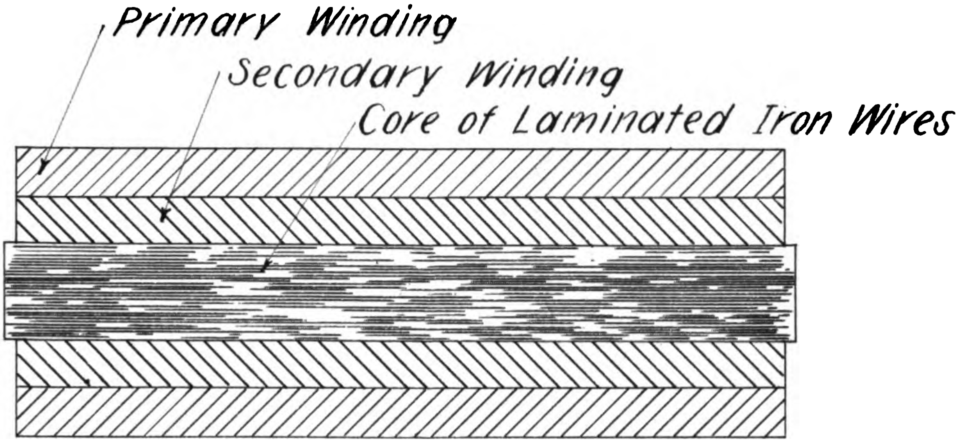
Figure Number	Calibration curve Number.	Angle of Lag Deg.	Total current of 20 receivers Amps.	Current through voltage vibrators Amps.	Resistance of voltage vibrator circuit. Ohms.	Resistance of current vibrator plus lead #6. See Fig. 3.	Voltage across current vibrator lead #6 & Receivers.
11	1	10°48'	.0514	.0561	7.3	.73	.41
--	1	19°45'	.1053	.1017	9.05	.73	.92
12	1	19°12'	.1053	.1030	9.05	.73	.932
--	1	18°9'	.0659	.0771	9.05	.73	.698
13	1	24°54'	.0626	.0738	9.05	.73	.668

Calculated Data- Continued.

Total impedance. Ohms.	Total Resistance. Ohms.	Resistance of 20 receivers in parallel. Ohms.	Reactance of 20 receivers in parallel. Ohms.	Resistance of one receiver. Ohms.	Inductance of onee receiver. Millihenrys	Frequency.
	4.35	D.C. Resistance		87.0		D.C.
7.98	7.84	7.11	1.501	142.2	77.8	61.5
8.72	8.20	7.47	2.950	149.4	53.6	175.5
8.85	8.36	7.63	2.910	152.6	52.8	175.5
10.92	10.40	9.67	3.482	193.4	21.08	526.5
10.68	9.67	8.94	4.490	178.8	27.2	526.5

Fig.-14

Western Electric Induction Coil
No. 20

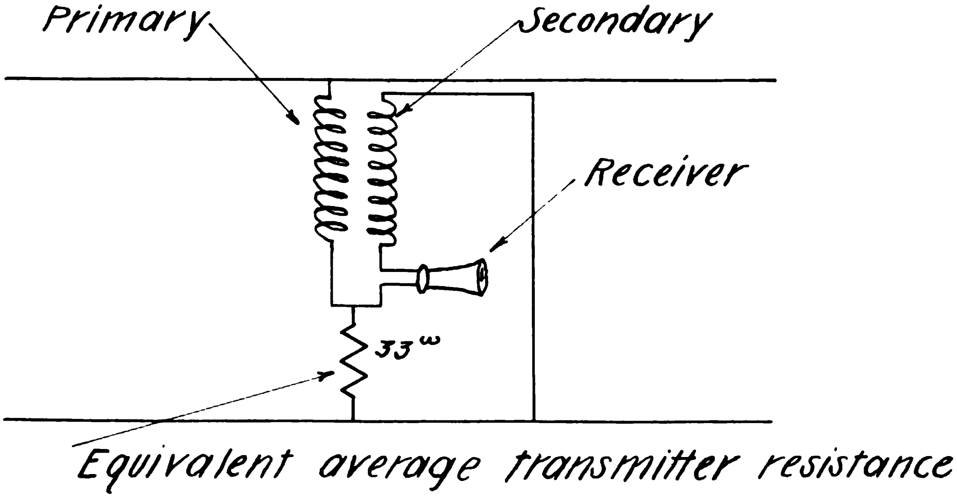


	D.C. Resistance	No. of turns	Diameter of wire
Primary	16.1 ohms	1700	0.0158"
Secondary	26 ohms	1400	0.00589"

Length of core - $4\frac{1}{8}$"   Weight of core - $47\frac{1}{2}$ gr.

Fig.-15

Telephone Connection used in
Oscillograph Measurements



Frequency - 62

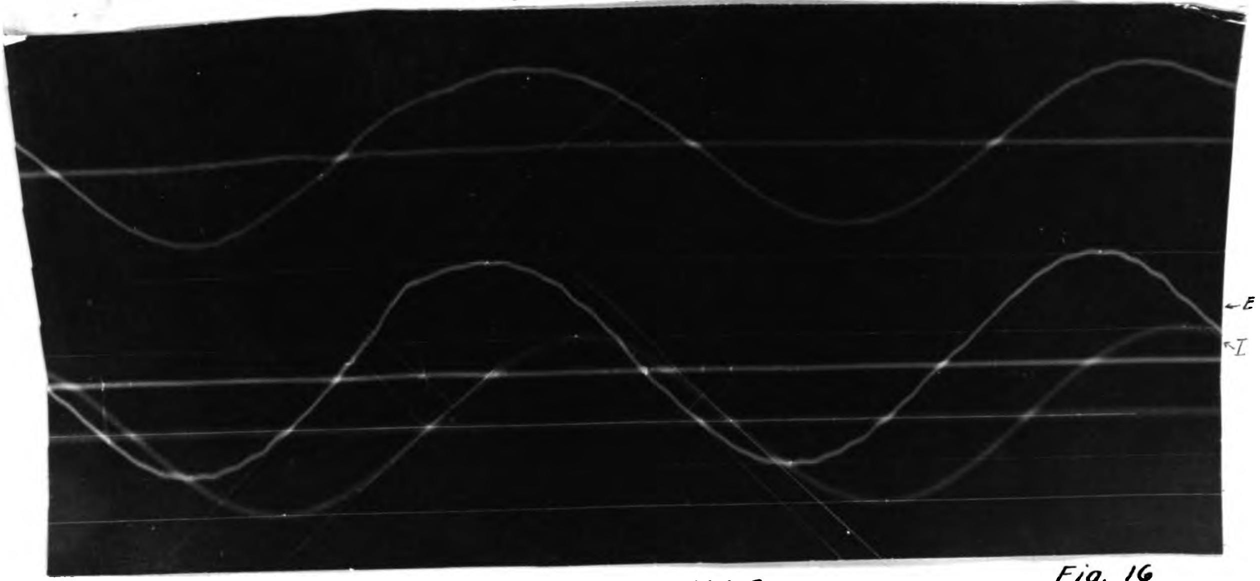


Fig. 16

Frequency - 166.6

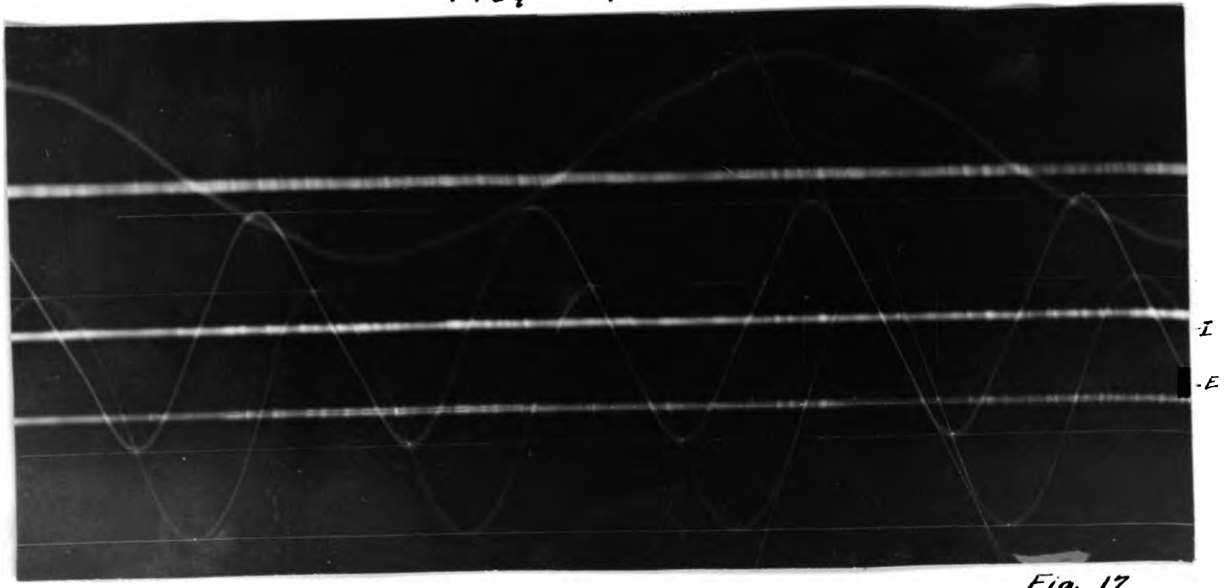
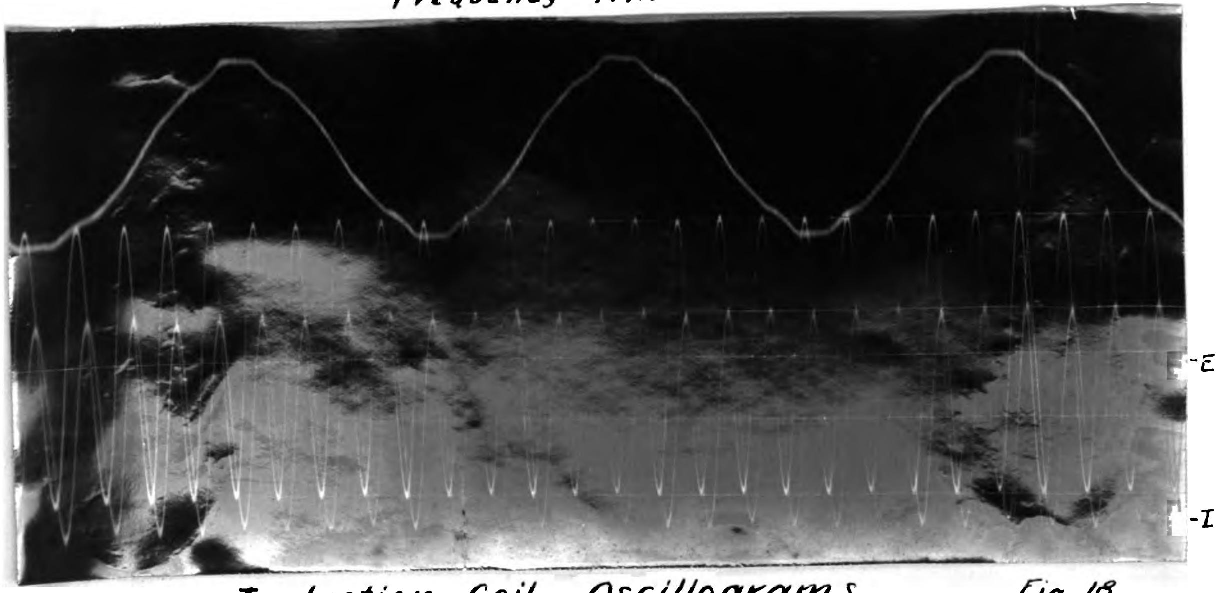


Fig. 17

Frequency - 499.6



Induction Coil Oscillograms

Fig. 18

Resistance and Inductance Determinations

on the Primary of

Western Electric Induction Coils, Catalogue No.20.

Oscillogram Measurements.

Figure Number.	Calibration curve Number.	Double deflection current vibrator In.	Double deflection voltage vibrator In.	Length of 360° Inches	Angle θ Deg.	a Inches	b Inches	Frequency. Cycles per Sec.
16	2	.86	1.04	3.07	$44^\circ 20'$	.505	.125	62
--	2	.88	1.06	3.08	$39^\circ 45'$	.55	.13	62.5
--	3	1.27	1.225	1.34	$69^\circ 10'$	.42	109	166.5
17	3	1.27	1.21	1.41	$68^\circ 40'$	.44	.10	166.5
18	2	1.11	1.425	0.22	$86^\circ 55'$	.06	.828	499.5
--	2	1.15	1.625	0.48	$84^\circ 30'$	.127	.055	499.5
--	2	1.6	2.175	0.50	85 25'	.13	.062	499.5

Calculated Data.

Figure Number.	Calibration curve Number	Angle of lag Degrees	Total current of 20 coils in parallel Amps.	Current through voltage vibrator. Amps.	Resistance of voltage vibrator circuit. Ohms.	Resistance of current vibrator plus lead #6 Ohms. Fig. 3.	Voltage across current vibrator, lead #6 and coils.
16	2	52°54'	.0533	.049	3.1	.73	.152
--	2	62°00'	.0546	.0499	3.1	.73	.1548
--	3	73°18'	.081	.0654	8.6	.73	.562
17	3	74°00'	.081	.0647	8.6	.73	.556
18	2	72°6'	.0689	.0671	14.57	.73	.978
--	2	68°36'	.0714	.0766	14.57	.73	1.115
--	2	69°40'	.0982	.1025	14.57	.73	1.492

Calculated Data Continued.

Total impedance Ohms.	Total resistance Ohms.	Resistance of 20 coils in parallel. Ohms.	Reactance of 20 coils in paralel. Ohms.	Resistance of one coil Ohms.	Inductance of one coil Millihenrys	Frequency
		.805	D.C Resistance	16.1		D.C.
2.85	1.72	.99	2.24	19.8	114.8	62
2.84	1.331	.601	2.51	12.02	127.6	62.5
6.94	2.11	1.38	6.65	27.6	127.2	166.5
6.86	1.887	1.157	6.6	23.14	126.2	166.5
14.2	4.36	3.63	13.5	72.6	86.2	499.5
15.6	5.69	4.96	14.54	99.2	92.8	499.5
15.2	5.28	4.55	14.26	91.0	91.0	499.5

Frequency - 61.5

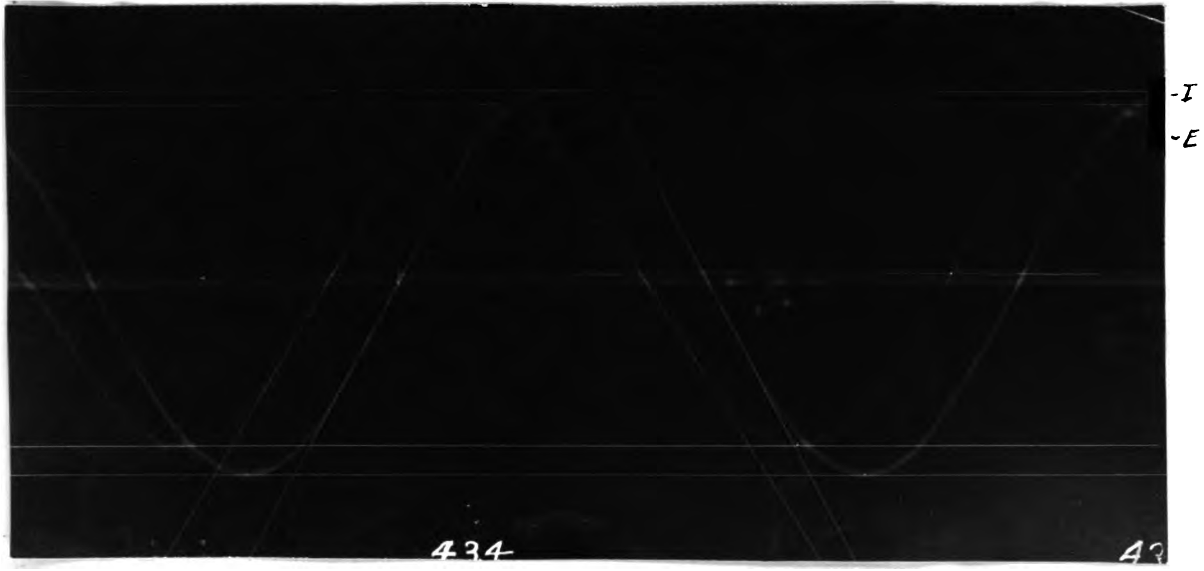


Fig. 19

Frequency - 166.5

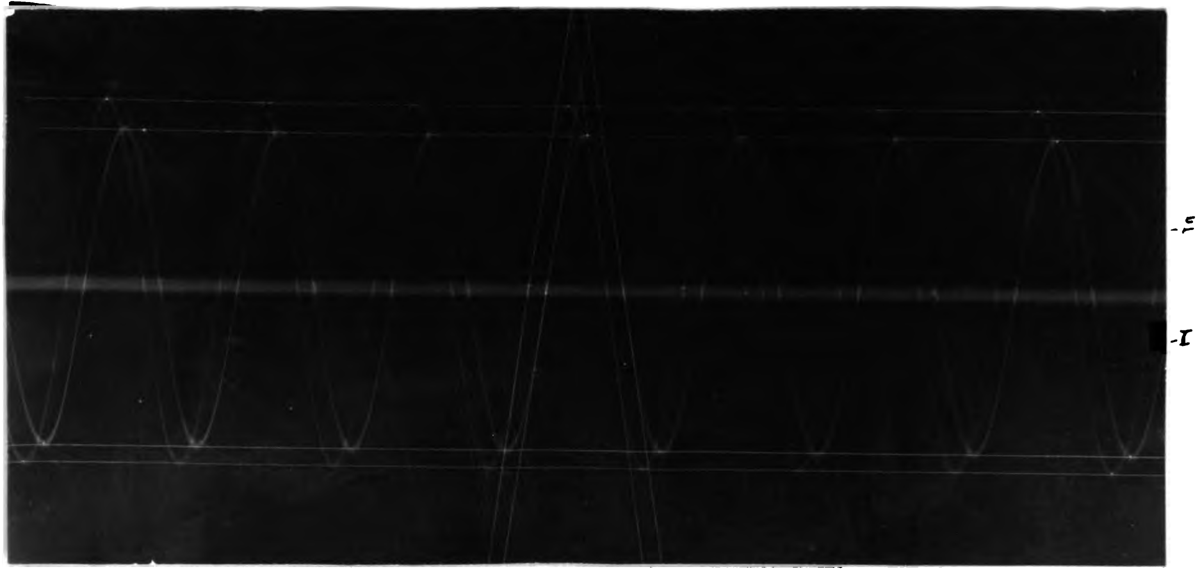


Fig. 20

Frequency - 499.5

Telephone Oscillograms. Fig. 21

Resistance and Inductance Determinations on Telephone Connection as per Fig. 15.

Oscillogram Measurements.

Figure Number	Calibration curve Number	Double deflection current vibrator In.	Double deflection voltage vibrator In.	Length of 360° Inches	Angle θ Degrees	a Inches	b Inches	Frequency. Cycles per Sec.
19	4	1.975	1.750	3.22	60°00'	.370	.280	61.5
20	4	1.620	1.860	0.81	82°00'	.095	.085	166.5
--	4	1.620	1.885	1.20	78°00'	.140	.125	166.5
21	4	1.565	1.490	0.50	84°50'	.600	.400	499.5

Calculated Data.

Figure Number	Calibration curve Number.	Angle of Lag Degrees	Total current of 20 telephone sets. Amps.	Current through voltage vibrator Amps.	Resistance of voltage vibrator circuit. Ohms.	Resistance of current vibrator & lead #6 See Fig. 3 Ohms.	Voltage across current vibrator, lead #6 and telephone sets.
19	4	41°50'	.1235	.0945	6.56	.73	.619
20	4	40°20'	.1014	.1004	9.56	.73	.960
--	4	40°30'	.1010	.1020	9.56	.73	.976
21	4	36°12'	.0980	.0805	19.56	.73	1.570

Calculated Date Continued.

Total impedance Ohms.	Total resistance Ohms.	Resistance of 20 telephone sets in paralel Ohms.	Reactance of 20 telephone sets in parallel Ohms.	Resistance of one telephone set Ohms.	Inductance of one telephone set Millihendrys	Frequency cycles.
5.01	3.73	3.00	3.34	60.00	173.0	61.5
9.46	7.21	6.48	6.12	129.6	117.0	166.5
9.66	7.34	6.61	6.27	132.2	119.6	166.5
16.05	12.97	9.45	12.24	244.8	61.6	499.5

Alternating Current Bridge Measurements.

As an illustration of the methods used in calculating the resistance and inductance of the apparatus by means of data obtained with an A.C. bridge, consider the first determination of the receiver resistance and inductance. Using bridge A in Fig. 4 a balance was obtained by varying the resistance and capacity. This resistance was found to be 2050 ohms and the capacity was .149 micro farads. Then the resistance of the receiver is equal to $\frac{250000}{2050} = 122$ ohms.

The inductance in milli-henrys is

$$L = 250\ C$$

where C is in micro-farads. Then

$$L = 250 \times .149$$

$$= 37.3 \text{ milli-henrys.}$$

The frequency was determined by means of the auxiliary bridge B

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$L = .1 \text{ henry}$$

$$f = \frac{504}{\sqrt{C}}$$

where C is in micro-farads.

In a similar manner the resistance and inductance of the induction coils and telephone sets were determined.

Calculated Results from Bridge Measurements.

Western Electric Receiver Catalogue No.143 A.W.

Variable Resistance Ohms.	Variable capacity micro-farads.	Resistance of Receivers, Ohms.	Inductance of Receiver, Milli-henrys	Frequency in Cycles.
2050	.149	122	37.3	572
1870	.132	134	33.0	654
1590	.126	157	31.5	845
1570	.110	159	27.5	1280
1425	.107	175	26.8	1520
1400	.101	179	25.2	1635

Western Electric Induction Coils Catalogue No. 20.

Variable Resistance Ohms.	Variable capacity micro-farads.	Resistance of one coil in Ohms.	Inductance of one coil milli-henrys	Frequency.
8550	.508	29.2	128	564
7450	.508	33.6	128	716
7000	.508	35.7	128	776
5700	.507	43.9	127	1060
4920	.505	50.9	126	1240
3720	.487	67.2	122	1635

Western Electric Telephone Sets.

Variable Resistance Ohms.	Variable capacity micro-farads.	Resistance of telephone set Ohms.	Inductance of telephone set Milli-henrys.	Frequency
790	.24	316	60.0	564
755	.212	340	53.0	716
735	.204	340	51.0	776
705	.154	355	38.5	1060
675	.149	370	36.3	1240
580	.130	430	32.5	1635

Data for Plotting Curves of Resistance and Inductance against Frequency.

Western Electric Receivers Catalogue No.143 A.W.

Frequency Cycles/sec.	Resistance-Ohms	Inductance-milli-henryes	
61.5	142.2	77.8	
175.5	151.0	53.2	Oscillogram Determinations
526.5	186.1	24.24	
572.0	122.0	37.3	
654.0	134.0	33.0	
845.0	157.0	31.5	Bridge Determinations.
1280.0	159.0	27.5	
1520.0	175.0	26.8	
1635.0	179.0	25.2	

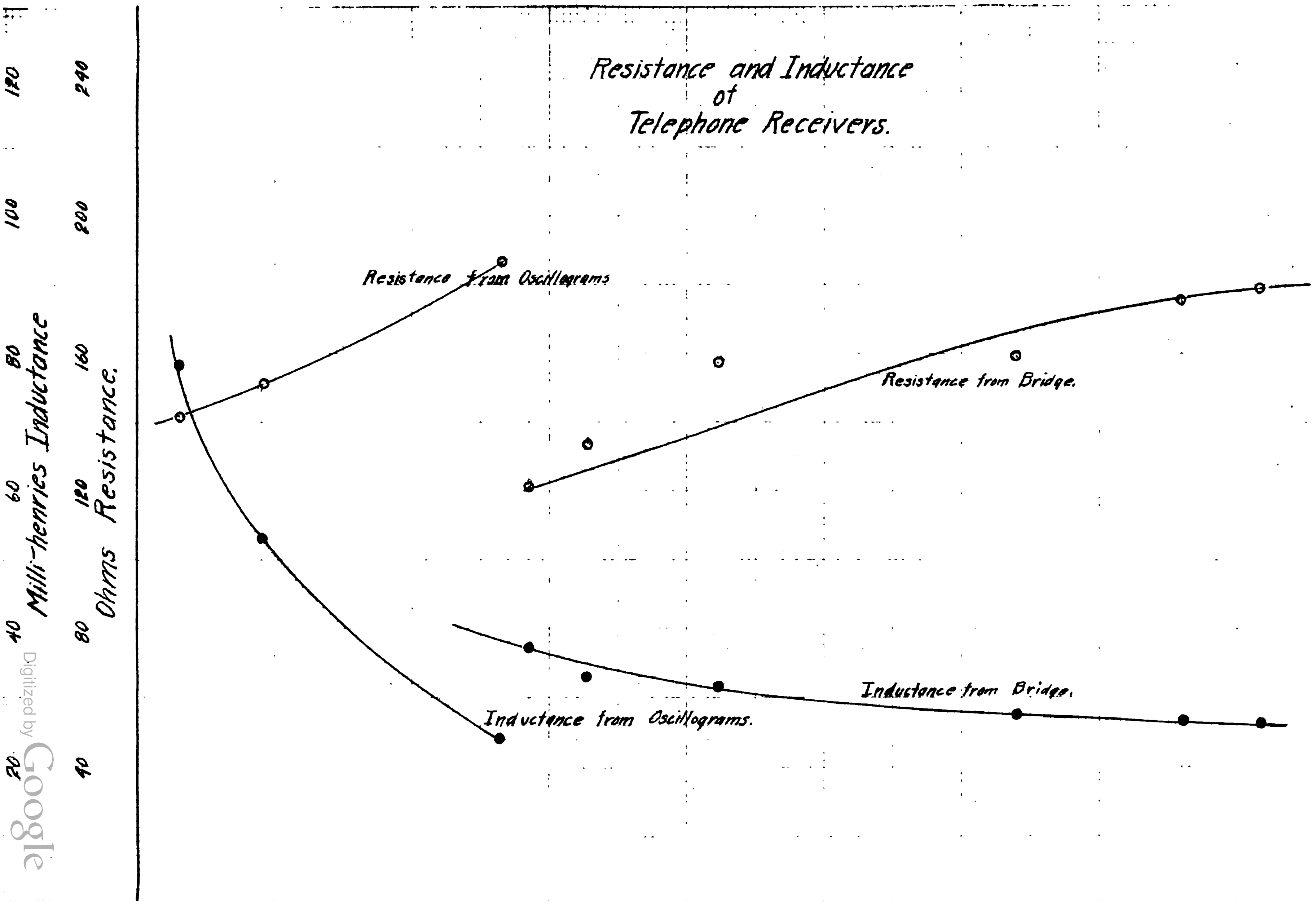
Western Electric Induction Coils. Catalogue No.20

Frequency cycles/sec.	Resistance-Ohms.	Inductance-milli-henrys	
62.0	15.91	121.2	
166.5	25.37	126.7	Oscillogram Determinations
499.5	87.6	90.0	
564.0	29.2	128.0	
716.0	33.6	128.0	
776.0	35.7	128.0	Bridge Determinations.
1060.0	43.9	127.0	
1240.0	50.9	126.0	
1635.0	67.2	122.0	

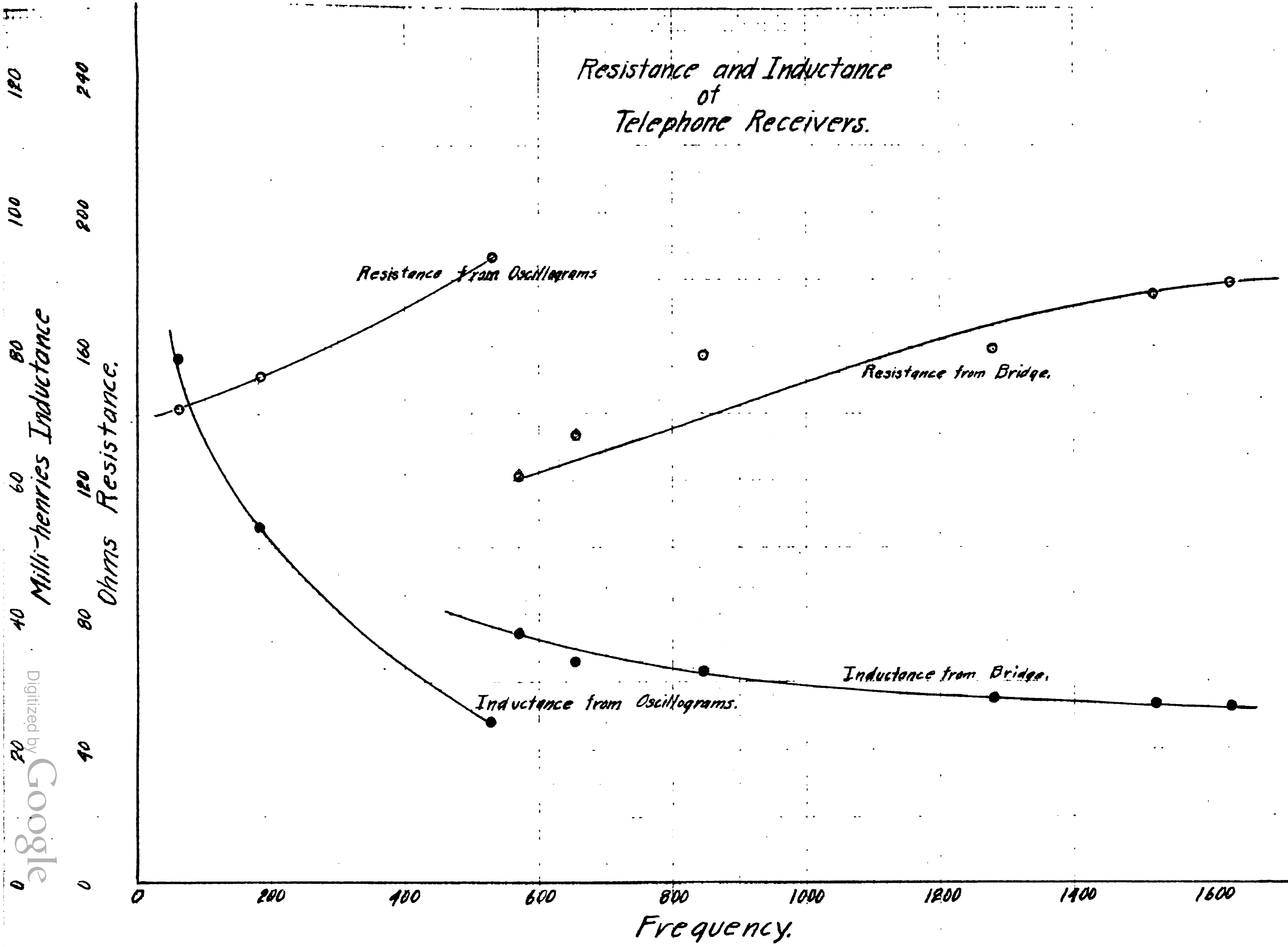
Western Electric Telephone Sets.

Frequency cycles/sec.	Resistance-Ohms.	Inductance - Milli-henrys	
61.5	60.0	173.0	
166.5	130.9	118.3	Oscillogram Determinations
499.5	244.8	61.6	
564.0	316.0	60.0	
716.0	340.0	53.0	
776.0	340.0	51.0	Bridge Determinations
1060.0	355.0	38.5	
1240.0	370.0	36.3	
1635.0	430.0	32.5	

Fig. 22



Resistance and Inductance of Telephone Receivers.



Resistance and Inductance of Telephone Receivers.

Fig. 22

Fig. 23

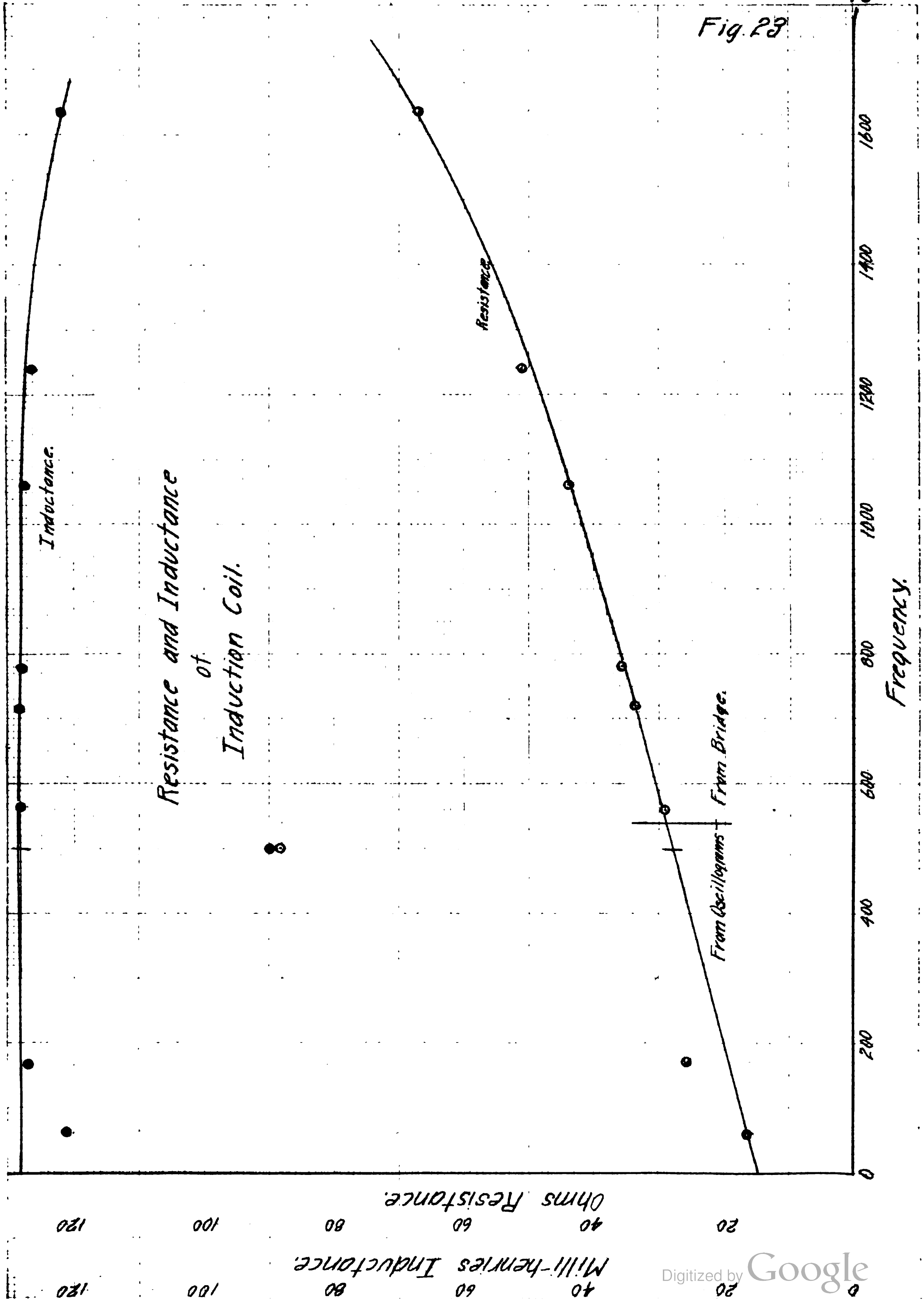
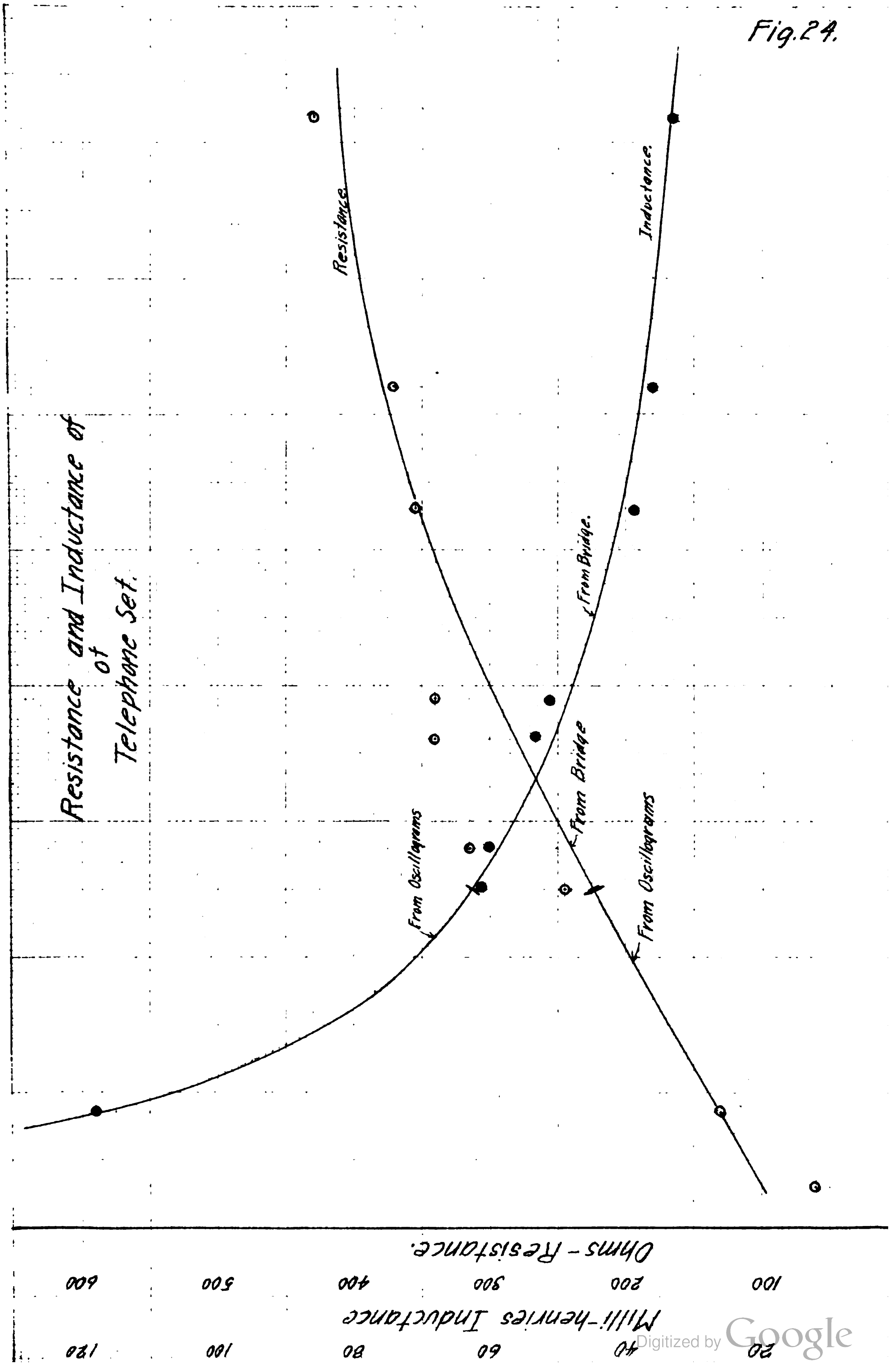

Resistance and Inductance
of
Induction Coil.
Inductance.
Resistance
From Bridge.
From Oscillograms
Frequency.
0
200
400
600
800
1000
1200
1400
1600
Ohms Resistance.
20
40
60
80
100
120
Milli-henries Inductance.
0
20
40
60
80
100
120

Fig.24.


Resistance and Inductance of Telephone Set.
Resistance
Inductance.
From Bridge.
From Bridge
From Oscillograms
From Oscillograms
Ohms - Resistance.
100
200
300
400
500
600
Milli-henries Inductance
20
40
60
80
100
120

Data for Plotting Current and Voltage Curves of a Telephone Line at Various Frequencies.

Line Data-

Two overhead wires spaced 10" apart. No. 12 New British Standard conductors.

Diameter of Wire - .104"

R = 1.943 ohms per 1000' loop.

Resistance from handbook.

q = zero (assumption)

$$L = .01524 + .14 \log_{10} \frac{D}{r}$$

where D - distance between wires.

r = radius of one conductor.

$$L = .01524 + .14 \log_{10} \frac{10}{\frac{.104}{2}}$$

$$= .01524 + .14 \times 2.2833$$

= .335 Milli-henrys per 1000' wire.
= .670 Milli-henrys per 1000' loop

$$C = \frac{.003677}{\log_{10} \frac{D}{r}}$$

$$= \frac{.003677}{2.2833} = .00161 \text{ micro-farads per 1000' loop}$$

At 800 cycles per second

$$x = 2\pi fL = \frac{2\pi 800 \times .67}{1000}$$

= 3.36 ohms inductive reactance of 1000' loop

$$b = 2\pi fC = \frac{2\pi\, 800 \times .00161}{10^6}$$

$$= 8.08 \times 10^{-6} \text{ mhos}$$ susceptance per 1000' loop.

$$z = \sqrt{R^2 + x^2}$$

$$= \sqrt{(1.943)^2 + (3.36)^2}$$

$$= 3.88$$ ohms impedance per 1000' loop.

$$y = \sqrt{q^2 + b^2}$$

$$= \sqrt{0 + (8.08 \times 10^{-6})^2}$$

$$= 8.08 \times 10^{-6}$$ mhos admittance per 1000' loop

$$\alpha = \sqrt{\tfrac{1}{2}(zy + Rq - xb)}$$

$$= \sqrt{\tfrac{1}{2}(3.88 \times 8.08 \times 10^{-6} + 1.943 \times 0 - 3.36 \times 8.08 \times 10^{-6})}$$

$$= 10^{-3}\sqrt{\tfrac{1}{2}(3.88 \times 8.08 - 3.36 \times 8.08)}$$

$$= 1.45 \times 10^{-3}$$

$$\beta = \sqrt{\tfrac{1}{2}(zy - Rq + xb)}$$

$$= \sqrt{\tfrac{1}{2}(3.88 \times 8.08 \times 10^{-6} - 1.943 \times 0 + 3.36 \times 8.08 \times 10^{-6})}$$

$$= 10^{-3}\sqrt{\frac{8.08}{2}(3.88 + 3.36)}$$

$$= 5.4 \times 10^{-3}$$

$$h = \frac{1}{y}\sqrt{\tfrac{1}{2}(zy + Rq + xb)}$$

$$= \frac{10^6}{8.08}\sqrt{\tfrac{1}{2}(3.88 \times 8.08 \times 10^{-6} + 1.943 \times 0 + 3.36 \times 8.08 \times 10^{-6})}$$

$$= \frac{10^3}{8.08}\sqrt{\frac{8.08}{2}(3.88 + 3.36)}$$

$$= 668$$

$$K = \frac{1}{y}\sqrt{\tfrac{1}{2}(zy - Rq - xb)}$$

$$= \frac{10^6}{8.08}\sqrt{\tfrac{1}{2}(3.88 \times 8.08 \times 10^{-6} - 1.943 \times 0 - 3.36 \times 8.08 \times 10^{-6})}$$

$$= \frac{10^3}{8.08}\sqrt{\tfrac{1}{2} \times 8.08(3.88 - 3.36)}$$

$$= 179$$

Terminal Constants or Resitance and Inductance of Telephone Set across the end of the line.

From Curve, Fig. 24 at 800 cycles.

$r_1 = 300$ ohms

$L_1 = 48$ milli-henrys

$x_1 = 2\pi f L_1$

$= \frac{2\pi 800 \times 48}{1000}$

$= 241$ ohms inductive reactance.

Constant Terms of Current and Voltage Equations at 800 Cycles.

$$I = \frac{I_o}{2}\left\{1 + \frac{(r_1 + jx_1)}{(h + jK)}\right\} e^{\alpha l}(\cos \beta l + j\sin \beta l) +$$

$$\frac{I_o}{2}\left\{1 - \frac{(r_1 + jx_1)}{(h + jK)}\right\} e^{-\alpha l}(\cos \beta l \div j\sin \beta l)$$

$$E = \frac{I_o}{2}\left\{(r_1 + jx_1) + (h + jK)\right\} e^{\alpha l}(\cos \beta l + j\sin \beta l) +$$

$$\frac{I_o}{2}\left\{(r_1 + jx_1) - (h + jK)\right\} e^{-\alpha l}(\cos \beta l - j\sin \beta l)$$

$$\left\{1 + \frac{(r_1 + jx_1)}{(h + jK)}\right\} = \left\{1 + \frac{(300 + j241)}{(668 + j179)}\right\}$$

$$= \left\{1 + \frac{(300 \times 668 + 241 \times 179) - j(300 \times 179 - 241 \times 668)}{(668)^2 + (179)^2}\right\}$$

$$= 1 + \frac{(247200) + j(108300)}{478000} = (1 + .517 + j.227)$$

$$= (1.517 + j.227)$$

$$\left\{1 + \frac{r_1 + jx_1}{L + jK} = (1 - .517 - j.227)\right.$$

$$= (.483 - j.227)$$

$$(r_1 + jx_1) + (L + jK) = (300 + j241) + (668 + j179)$$

$$= (968 + j420)$$

$$(r_1 + jx_1) - (L + jK) = (-368 + j62)$$

In a similar manner these constants are computed for other frequencies.

Table of Constants per 1000' loop Distance.

Frequency	800	1000	1200	1400	1600
R - Ohms	1.943	1.943	1.943	1.943	1.943
q - mhos	0	0	0	0	0
L - mh	.67	.67	.67	.67	.67
C - m.f.	.00161	.00161	.00161	.00161	.00161
x - ohms	3.36	4.2	5.04	5.88	6.72
b - mhos	$8.08\text{x}10^{-6}$	$10.1\text{x}10^{-6}$	$12.12\text{x}10^{-6}$	$14.15\text{x}10^{-6}$	$16.17\text{x}10^{-6}$
z - ohms	3.88	4.62	5.40	6.19	6.995
y - mhos	$8.08\text{x}10^{-6}$	$10.1\text{x}10^{-6}$	$12.12\text{x}10^{-6}$	$14.15\text{x}10^{-6}$	$16.17\text{x}10^{-6}$
α -	$1.45\text{x}10^{-3}$	$1.455\text{x}10^{-3}$	$1.475\text{x}10^{-3}$	$1.48\text{x}10^{-6}$	$1.495\text{x}10^{-6}$
β	$5.4\text{x}10^{-3}$	$6.68\text{x}10^{-3}$	$7.96\text{x}10^{-3}$	$9.26\text{x}10^{-3}$	$10.53\text{x}10^{-6}$
h -	668	660	656	654	651
K -	179	144	122	104.5	92.2
r_1 - ohms	300	345	380	400	413
L_1 - m.h.	48	42	38	35.5	33.5
x_1 - ohms	241	264	286	312	336
$1+\frac{r_1+jx_1}{h+jK}$	1.517-j.227	1.583-j.273	1.636-j.317	1.671-j.370	1.694-j.418
$1-\frac{r_1+jx_1}{h+jK}$	.483-j.227	.417-j.273	.364-j.317	.329-j.370	.306-j.418
$(r_1+jx_1)+(h+jK)$	968-j420	1005-j408	1036-j408	1054-j416.5	1064-j428.2
$(r_1+jx_1)-(h+jK)$	-368-j62	-315-j120	-276-j164	-254-j207.5	-238-j243.8

Currents and Voltages along the Telephone Line
at 800 cycles per second.

Assume the current at the receiving end of the line,

$$\underset{\cdot}{I}_o = .0002 + j0$$

When 1 = 0, at the receiving end of the line, the incoming current is expressed,

$$\underset{\cdot}{I}'_o = \frac{\underset{\cdot}{I}_o}{2}\left\{1 + \frac{(r_1 + jx_1)}{(h + jK)}\right\} = .0001(1.517 + j.227)$$

$$= .0001517 + j.0000227$$

The absolute value of the incoming current equals,

$$\sqrt{(.0001517)^2 + (.0000227)^2}$$

$$\underset{\cdot}{I}'_o = .0001535 \text{ amperes}$$

The reflected current is

$$\underset{\cdot}{I}''_o = \frac{\underset{\cdot}{I}_o}{2}\left\{1 - \frac{(r_1 + jx_1)}{(L + jK)}\right\}$$

$$= .0001(.483 - j.227)$$

$$= .0000483 - j.0000227$$

$$= .0000533 \text{ amperes.}$$

The voltage causing the incoming current to flow when 1 = 0 is

$$\underset{\cdot}{E}'_o = \frac{\underset{\cdot}{I}_o}{2}\left\{(r_1 + jx_1) + (L + jK)\right\}$$

$$E'_o = .0001\ (968 + j420)$$
$$= .0968 + j.042$$
$$= .1056 \text{ volts.}$$

The voltage causing the reflected current to flow when $l = 0$ is

$$E''_o = \frac{I_o}{2}\left\{(r_1 + jx_1) - (L + jK)\right\}$$
$$= .0001(-368 + j62)$$
$$= .0372 \text{ volts}$$

The vector expressions for the currents and voltages give the phase position at $l = 0$. Then since βl is chosen in 30° increments the phase position of the succeeding incoming current and voltages will be 30° in advance of the initial current and voltage because of the term $(\cos \beta l + j \sin \beta l)$. The succeeding reflected current and voltage will lag 30° behind the intial reflected current and voltage because of the term $(\cos \beta l - j\sin \beta l)$.

The absolute values of the incoming current and voltage is determined by taking the product of the absolute value at $l = 0$ and the term $e^{\alpha l}$, l being determined from $\beta l = 30^\circ$. 60°, etc Similarly the reflected current and voltage is product of initial value and $e^{-\alpha l}$.

Data for Current and Voltage Diagrams at 800 Cycles.

$$\dot{I}_o = .0002 + j0$$

Initial Values 1 = 0

Vector Expressions.	Absolute Values.
$\dot{I}'_o$ =.0001517 + j.0000227	$\dot{I}'_o$ = .0001535 amperes.
$\dot{E}'_o$ = .0968 + j.042	$\dot{E}'_o$ = .1056 volts
$\dot{I}''_o$ = .0000483 - j.0000227	$\dot{I}''_o$ = .0000533 amperes
$\dot{E}''_o$ = -.0368 + j.0062	$\dot{E}''_o$ = .0372 volts

Number	1 Degrees	Distance from receiving end of line 1000 ft.	$e^{\alpha 1}$	$e^{-\alpha 1}$	I' Amps. Incoming.	I" Amps. Reflected.	E' Volts. Incoming.	E" Volts Reflected
0	0	0	1	1	.0001535	.0000533	.1056	.0372
1	30	96.9	1.151	.868	177	463	.122	323
2	60	193.8	1.328	.754	204	396	.140	281
3	90	291	1.523	.656	234	350	.161	244
4	120	388	1.759	.569	269	303	.186	212
5	150	485	2.029	.493	312	262	.214	183
6	180	582	2.344	.427	361	228	.248	159
7	210	678	2.700	.372	415	196	.285	139
8	240	776	3.118	.321	478	171	.329	119
9	270	874	3.595	.281	553	150	.379	105
10	300	969	4.160	.241	638	127	.440	.0089
11	330	1068	4.800	.208	737	111	.506	77
12	360	1163	5.540	.181	852	.0000096	.585	67

Data for Current and Voltage Diagrams at 1000 Cycles.

$I_{.0} - .0002 + j0$

Initial Values 1 = 0

Vector Expressions	Absolute Values.
$I'_{.0} = .0001583 + j.0000273$	$I'_{.0} - .0001606$ amperes
$E'_{.0} = .1005 + j.0408$	$E'_{.0} = .1084$ volts
$I''_{.0} - .0000417 - j.0000273$	$I''_{.0} = .0000498$ amperes
$E''_{.0} = -.0315 + j.012$	$E''_{0} - .0337$ volts

Number	1 Degrees	Distance receiving end of line 1000 ft.	$e^{\alpha l}$	$e^{-\alpha l}$	I' Amperes incoming.	I" Amperes reflected.	E' Volts Incoming	E" Volts reflected
0	0	0	1	1	.0001606	.0000498	.1084	.0337
1	30	78.3	1.121	.891	1805	444	.1328	. 300
2	60	156.5	1.258	.794	202	395	.1487	267
3	90	234.8	1.411	.706	226	353	.1670	238
4	120	313.0	1.582	.631	254	315	.1873	213
5	150	391	1.773	.563	285	281	.2100	189
6	180	469	1.988	.502	319	251	.2350	169
7	210	547	2.230	1446	358	223	.2660	150
8	240	626	2.505	.399	403	199	.2965	135
9	270	704	2.810	.355	452	177	.3330	119
10	300	783	3.150	.327	506	163	.3725	110
11	330	860	3.530	.282	567	141	.4170	.0095
12	360	938	3.960	.252	636	126	.4680	85

Data for Current and Voltage Diagrams at 1200 Cycles.

$I_{.0} = .0002 + j0$

Initial Values $l = 0$

Vector Expressions	Absolute Values.	
$I'_{.0} = .0001636 + j.0000317$	$I'_0 = .0001667$	amperes.
$E'_{.0} = .1036 + j.0408$	$E'_0 = .1113$	volts
$I''_{.0} = .0000364 - j.0000317$	$I''_0 = .00004825$	amperes.
$E''_{.0} = .0276 + j.0164$	$E''_0 = .0321$	volts.

Number	l Degrees	Distance from receiving end of line 1000 ft.	$e^{\alpha l}$	$e^{-\alpha l}$	I' Amperes incoming	I" Amperes reflected	E' Volts Incoming	E" Volts reflected.
0	0	0	1	1	.0001667	.00004825	.1113	.0321
1	30	65.8	1.102	.907	184	437	.1245	292
2	60	131.6	1.212	.824	202	397	.137	265
3	90	197.5	1.336	.746	223	361	.151	241
4	120	263.8	1.462	.677	245	328	.165	218
5	150	329.0	1.624	.615	271	297	.183	198
6	180	395.5	1.789	.557	298	269	.202	176
7	210	461.0	1.971	.506	329	244	.223	163
8	240	526.0	2.170	.459	362	221	.245	148
9	270	593.0	2.395	.417	398	202	.271	134
10	300	658.0	2.635	.378	439.0	183	.300	122
11	330	724.0	2.900	1344	484	167	.328	111
12	360	789.0	3.195	.311	533	150	.349	100

Data for Current and Voltage Diagrams at 1400 Cycles.

$\underset{\cdot}{I}_o = .0002 + j0$

Initial Values $l = 0$

Vector Expressions.	Absolute Values.	
$\underset{\cdot}{I}'_o = .0001671 + j.000037$	$I'_o = .000171$	amperes
$\underset{\cdot}{E}'_o = .1054 + j.04165$	$E'_o = .1134$	volts
$\underset{\cdot}{I}''_o = .0000329 - j.000037$	$I''_o = .0000495$	amperes.
$\underset{\cdot}{E}''_o = -.0254 + j.02075$	$E''_o = .0328$	volt

Number.	l Degrees	Distance from receiving end of line 1000 ft.	$e^{\alpha l}$	$e^{-\alpha l}$	I' Amperes. incoming.	I" Amperes reflected.	E' Volts incoming	E" Volts reflected.
0	0	0	1	1	.000171	.0000495	.1134	.0328
1	30	56.6	1.087	.921	186	456	.123	302
2	60	113.2	1.180	.849	202	420	.134	278
3	90	170.0	1.280	.783	210	388	.145	257
4	120	226.4	1.390	.721	237	357	.158	237
5	150	283.0	1.510	.664	258	329	.171	218
6	180	340.0	1.640	.612	280	303	.186	201
7	210	396.0	1.780	.564	306	279	.202	185
8	240	454.0	1.940	.519	332	257	.220	170
9	270	510.0	2.110	.478	361	237	.239	157
10	300	566.0	2.290	.441	392	218	.259	144
11	330	625.0	2.480	.406	424	201	.282	133
12	360	680.0	2.700	.374	461	185	.306	122

Data for Current and Voltage Diagrams at 1600 Cycles.

$I_{\cdot o} - .0002 + j0$

Initial Values $l = 0$

Vector Expressions	Absolute Values.	
$I'_{\cdot o} = .0001694 + j.0000418$	$I'_o = .0001744$	amperes
$E'_{\cdot o} = .1064 + jx.04282$	$E'_o = .1147$	volts
$I''_{\cdot o} = .0000306 - j.0000418$	$I''_o = .0000518$	amperes
$E''_{\cdot o} = -.0238 + j.02438$	$E''_{\cdot o} = .03405$	volts.

Number	l Degrees	Distance from receiving end of line 1000 ft.	$e^{\alpha l}$	$e^{-\alpha l}$	I' Amperes. incoming	I" Amperes. reflected.	E' Volts incoming.	E" Volts reflected
0	0	0	1.000	1.000	.0001744	.0000518	.1147	.03405
1	30	49.7	1.077	.929	188	481	.124	316
2	60	99.4	1.158	.863	202	447	.133	294
3	90	149.1	1.243	.801	217	416	.142	273
4	120	198.9	1.340	.744	234	386	.154	253
5	150	248.5	1.440	.692	251	359	.165	235
6	180	298.0	1.555	.644	271	334	.178	219
7	210	348.0	1.675	.597	292	309	.192	203
8	240	398.0	1.805	.556	315	288	.207	189
9	270	447.5	1.940	.516	339	268	.222	176
10	300	497.0	2.090	.479	365	249	.239	163
11	330	546.0	2.250	.446	393	231	.258	152
12	360	596.0	2.420	.414	423	215	.278	141

CHAPTER V.

Conclusions.

Discussion of Curves and Results.

Figures 22, 23, and 24 of the preceding chapter, show how the resistance and inductance of various telephone apparatus changes with the frequency.

Fig. 22 shows the resistance and inductance curves for a Western Electric Receiver, No. 143 A.W. These curves are shown as broken but in reality they are continuous. The fact that they are broken is due to the different methods used in determining the points on the curve. The oscillograph was used in making the determinations at low frequencies and the bridge was used for the high frequencies. No satisfactory explanation presents itself for the discrepancy between the two methods of measurement. At any rate the curves bring out the fact that the resistance increases with the frequency and the inductance decreases with the frequency. The resistance, which is effective resistance, would be expected to increase as the frequency increased due to hysteresis and eddy current losses in the iron, which increase with the frequency. The inductance decreases due to the eddy currents set up in the magnet. These currents cause a flux which opposes the flux caused by the current in the conductor and hence decreases the amount of effective flux linking the windings. Since the inductance is directly proportional to the flux linking with the winding per unit current it can be seen that the inductance will decrease as the frequency increases

because the

eddy currents and hence also the flux set up by the eddy currents are directly proportional to the frequency.

Fig. 23 shows the resistance and inductance curves of Wester Electric Induction Coil No. 20, with secondary open. A coil of this type is shown diagrammatically in Fig. 14. An examination into the construction of the coil brought out the facts as presented on the figure. The core was about one-half inch in diameter and was made up of a bundle of annealled iron wires. The secondary winding was placed on the core and the primary winding over the secondary. The primary and secondary windings were separated from each other by a heavy paper, but the layers of secondary windings were in no way insulated from each other except by the cotton covering on the wire. However, in the primary winding every two layers were separated from the next two by several layers of paraffine paper. The number of turns on the primary and secondary were found to be 1700 and 1400 respectively. Thus the ratio of turns is as 1 to .824 .

The curves of Fig. 23 show that the resistance of the coil increases with the frequency as in the case of the receiver. However, the inductance curve shows the inductance to be very nearly independent of the frequency, which would be the case if the permeability of the iron remained constant and if the eddy current loss was zero. As stated above, the core of the coil is made up of a bundle of annealled iron wires. The wire core serves the same purpose as laminating and hence the eddy current loss is a minimum. Thus it would be expected that the inductance is almost independent of the frequency.

In this case the resistance and inductance at the low frequencies were determined by oscillogram measurements whereas for high frequencies the bridge was used. However, the discrepancy between the two methods of measurement is not as marked in this case as it was in the receiver determination.

Fig. 24, called Resistance and Inductance of a Telephone Set, are not the absolute correct curves of these values. In the first place the measurements made at low frequencies were with oscillograph and the telephone connections used were those shown in Fig. 15. There were twenty of these connections in parallel, so the average resistance and inductance of the connection was obtained. An artificial resistance of the same magnitude as the average transmitter resistance was substituted for the transmitter. Then this connection, Fig. 15, differs from a subscribers telephone set only in that the bell and condenser have been omitted from the circuit. The measurements at the high frequencies were made with the bridge and were made on a subscriber's telephone set which contained all the apparatus, receiver, induction coil, condenser, bell and transmitter. So from the manner in which the data was obtained for these curves it can be seen that the measurements made with the oscillograph might not be consistent with those obtained with the bridge. However, it happens that the curves obtained by both methods of measurement agree.very well so it must be that the bell and capacity in the telephone set have very little effect on the resistance and reactance of the whole set.

The second error is in the inductance curve as determined by the bridge. In the telephone set the condenser is connected in parallel with the secondary of the induction coil so in the bridge measurement the inductance determination is not one of true inductance but is a determination of some value which if multiplied by $2\pi f$ will give the reactance of the telephone set. However since it was only desired to know the resistance and reactance of the telephone set at various frequencies it was not deemed necessary to determine the exact inductance of the set.

Using the data obtained on the Western Electric Telephone Set, five diagrams were constructed. (See Tracings) These diagrams show the current and voltage relations in a long distance telephone line at frequencies of 800, 1000, 1200, 1400 and 1600 cycles per second. In each case the line conductors were No. 12 New British Standard, leakage was considered zero and a Western Electric Telephone Set was bridged across the end of the line.

To understand the use of the diagrams in determining the current and voltage in the line and the phase difference, consider the diagram for 1000 cycles. The primed symbols represent the incoming waves of current and voltage and the double primed symbols represent the reflected waves. The symbols which are not primed represent the total current and voltage in the line. In the construction of the diagrams the total current at the receiving end of the line was chosen as the reference vector. Now at the receiving end of the line the incoming wave of current

is represented by the vector marked I'_0 , and the reflected wave by the vector I''_0 . The angle between the vectors I'_0 and I''_0 is the phase difference between the incoming and reflected waves of current. Similarly at the receiving end of the line the incoming and reflected waves of voltage are represented by the vectors E'_0 and E''_0 respectively. The phase difference for the incoming waves of current and voltage is the angle between the vectors E'_0 and I''_0 and for the reflected waves the angle between E''_0 and I''_0 . Now consider the conditions at some point farther back on the line, say 74 miles from the receiving end of the line. From the table in the upper right hand corner of the sheet it is seen that this corresponds to point number 5. On the diagram the incoming and reflected waves of current are shown by vectors I'_5 and I''_5 and the corresponding voltages are E'_5 and E''_5. The phase relations between the incoming waves of current and voltage and the reflected waves of current and voltage are the same as at the end of the line. However, the current and voltage waves 74 miles from the receiving end of the line do not occupy the same phase position as at the end of the line. In a similar manner the current and voltage at any distance, up to one wave length, from the receiving end of the line can be determined. If the exact distance in miles is not given in the table the relations can be found by interpolation.

The total current and voltage at any point in the line can be determined as above. For example at the receiving end the current is I_0 and the voltage E_0. At this point the voltage leads the current by an angle of about 33°. Now at a point 74 miles

from the receiving end the current and voltage are I_5 and E_5 respectively. Hence the voltage leads by an angle of about 5°

From an inspection of all the diagrams it is seen that they all have the same general shape. The curves of the incoming and reflected waves are all pure lograithmic curves, but the curves of total current and voltage are irregular in form. This is to be expected since the curves were obtained by taking the vector sum of the incoming and reflected waves. Furthermore it will be noted that the higher the frequency the greater is the attenuation.

Now as stated in Chapter II, it is necessary to have waves of all frequencies attenuated proportionally and the reflected wave must be eliminated in order to obtain the best transmission of waves. It was also shown that if the values of the constants of the line were so proportioned that

$$L = \frac{CR}{q}$$

then attenuation of all frequency waves would be the same. It now remains to find a means of eliminating the reflected wave or at least to reduce this wave to a negligible quantity.

From an inspection of all the diagrams it is seen that the value of the reflected wave of current at the receiving end of the line differs very little for the different frequencies. The exact values are .0000533, .0000498, .00004825, .0000495 and .0000518 amperes for frequencies of 800, 1000, 1200, 1400 and 1600 cycles per second respectively. It will be remembered that

the initial values of the reflected waves of current were obtained by means of the expression

$$\frac{I_o}{2}\left\{1 - \frac{(r_1 + jx_1)}{(h + jK)}\right\}$$

If the factor $\frac{(r_1 + jx_1)}{(h + jK)}$ can be made equal to unity then the reflected wave will be entirely eliminated but since r_1, x_1, h and K are all dependent on the frequency the factor can not be made equal to unity except for one frequency.

In order that the feflected wave of the current be null the following expression must hold.

$$\left\{1 - \frac{(r_1 + jx_1)}{(h + jK)}\right\} = 0$$

Solving,

$$r_1 = h$$
$$x_1 = K$$

Now to apply these results to the line under consideration. From the diagrams it is seen that the reflected wave has the smallest initial value at 1200 cycles per second. This absolute value is .00004825 amperes. Under "Table of Constants" in the preceding chapter the values of r_1, x_1, h and K at 1200 cycles are,

r_1 -- 380 ohms
x_1 - 286 ohms
h = 656 ohms
K = 122 ohms

As shown above r_1, the terminal resistance must equal

h and x_1 , the terminal reactance, must equal K. Therefore there must be an additional resistance of

$$656 - 380 = 276 \text{ ohms}$$

placed across the end of the line.

The terminal reactance must be 122 ohms. Since x_1 is 286 ohms it is necessary to insert capacity reactance.

$$\frac{1}{2\pi fC} = 286 - 122 = 164 \text{ ohms.}$$

$$C = \frac{1}{2\pi 1200 \times 164} = .81 \text{ micro farads.}$$

Now if a resistance of 276 ohms at 1200 cycles and a condenser of .81 micro farads be inserted in series with the telephone set across the end of the line then the reflected wave will be entirely eliminated at 1200 cycles and will be small at other frequencies

The insertion of a resistance and capacity in series with the telephone set is, however, impracticable for two reasons. First the loss in the resistance would be so large that the total power transmitted would have to be greater. Second, the insertion of a capacity in series with the telephone limits the use of the telephone to a receiving station, since it will allow no D. C. current to pass and hence, cannot be used on common battery exchanges. Thus this method is impracticable and must be abandoned.

Since the insertion of a capacity in series with the receiving set has proved impractical, the only thing left is to change the terminal apparatus. Fig. 25 shows diagrammatically the usual telephone connections.

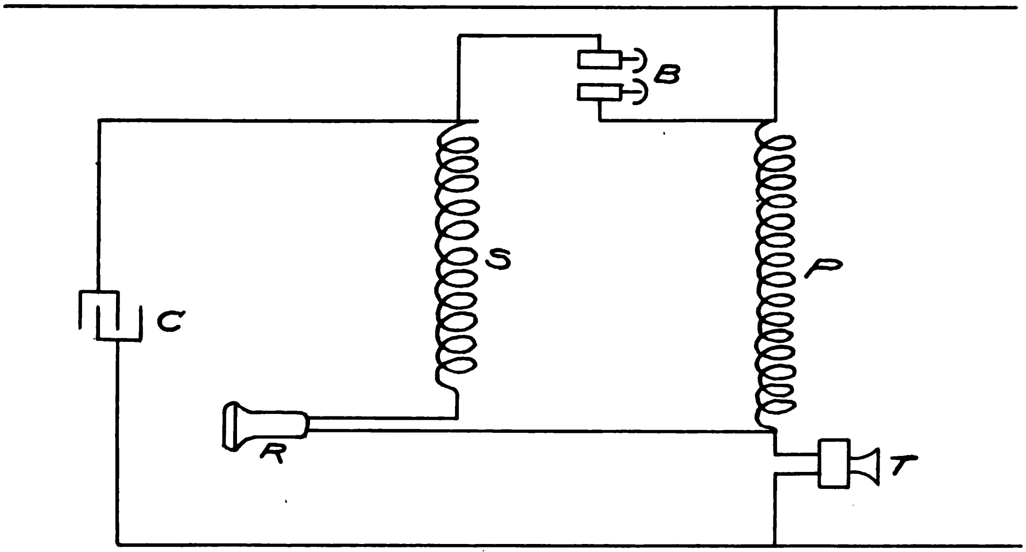


Fig. 25.

Referring to this figure, T is the transmitter with an average resistance of 30 ohms, R is the receiver whose resistance and inductance vary with the frequency, C is a 2 micro-farad condenser B is the ringer or signalling apparatus wound to 1000 ohms, P is the primary and S the secondary of an open core transformer, ratio 1 - .824. The connections are self explanatory, but a word might be said about the connection of the secondary of the transformer, usually called induction coil. This must be so inserted that the currents from the primary and secondary add up in the transmitter. Now if the transformer had such a ratio that the impedance of the telephone set equaled h + jk, the re-

flected waves would be zero. Suppose the open core transformer replaced by one which has a closed core and a very low exciting current. The impedance of the primary circuit would then be the impedance of the secondary multiplied by the square of the ratio of the number of turns plus the transmitter resistance. Expressed as an equation

$$\underset{\cdot}{z}p = \underset{\cdot}{z}_s \left(\frac{n_p}{n_s} \right)^2$$

An investigation into the conditions at 1200 cycles gives the following results:-

Receiver resistance - 152 ohms.
Receiver reactance - 206 ohms
h = 656 ohms
k = 122 ohms
Transmitter Resistance. = 30 ohms.

With no capacity in the secondary circuit.

Secondary impedance = 152 + j206 + 30
= 182 + j206 *

If transformer has 1 - 1 ratio the primary impedance is,

Primary impedance = 182 + 30 + j206 = 212 + j206

then the ratio of r_1 to h is

$$\frac{r_1}{h} = \frac{212}{656} = \frac{1}{3.25}$$

*Impedance of transformer assumed negligable.

and the ratio of x_1 to K

$$\frac{x_1}{K} = \frac{206}{122} = \frac{1}{.593}$$

The ratio of x_1 to K is thus seen to be approximately five times that of r_1 to h. Since it is desireable to have the ratios the same and equal to one if possible, it can readily be seen that such a condition is impossible with a one to one ratio of transformation. In order to make the ratio of r_1 to h equal to 1 the ratio of transformation of the impedance must be

$$\frac{656 - 30}{182} = 3.44$$

$$R_s\left(\frac{n_1}{n_2}\right)^2 + R_p = 656$$

$$\left(\frac{n_1}{n_2}\right)^2 = \frac{656 - R_p}{R_s}$$

$$\left(\frac{n_1}{n_2}\right)^2 = \frac{656 - 30}{182} = 3.44$$

Now to make the ratio of x_1 to K equal to 1, the secondary impedance must be

$$\frac{K}{3.44} = \frac{122}{3.44} = 35.5 \text{ ohms}$$

This requires a capacity in the secondary circuit whose reactance at 1200 cycles is,

$$206 - 35.5 = 170.5$$

$$\frac{10^6}{2\pi fC} - 170.5$$

$$C = \frac{10^6}{2\pi f \times 170.5}$$

$$C = \frac{10^6}{2\pi \times 1200 \times 170.5} \text{ micro-farads}$$

$$C = .778 \text{ micro-farads}$$

The impedance of the secondary circuit will then be,

$$\underset{.}{z}_s - (152 + 30) + j(206 - 170.5)$$
$$= 182 + j35.5$$

and the impedance of the primary is,

$$\underset{.}{z}_p -= 3.44(182 + j35.5) + 30$$
$$= 656 + j122$$

Then

$$I''_o = \frac{I_o}{2}\left\{1 - \frac{656 + j122}{656 + j122}\right\}$$

$$I''_o = 0$$

and

$$E''_o = \frac{I_o}{2}\left\{(656 + j122) - (656 + j122)\right\}$$

$$E''_o = 0$$

Hence the reflected waves are eliminated at 1200 cycles. If conditions are improved at 800 cycles and 1600 cycles, the changing of the transformers has added materially,

At 800 cycles.

Resistance of Receiver = 126 ohms

Reactance of Receiver = 154 ohms

h = 668 ohms

K = 179 ohms

Capacity Reactance = 255 ohms

Transmitter Resistance = 30 ohms

Then the secondary impedance,

$$Z_s = (126 + 30) + j(154 - 255)$$

$$= 156 - j101$$

Then

$$Z_p = 3.44(156 - j101) + 30$$

$$= 537 - j348 + 30$$

$$= 567 - j348$$

Then

$$I_0'' = \frac{I_0}{2}\left\{1 - \frac{567 - j348}{668 + j179}\right\}$$

$$= \frac{I_0}{2} \quad 1 - (0.66 - j.697)$$

$$= .0001\ (.34 + j.697)$$

$$= .000034 + j.0000697$$

$$I_0' = .0001\ (1 + .66 - j.697)$$

$$= .000166 - j.0000697$$

$$E_0'' = \frac{I_0}{2}\,(587 - j348) - (668 + j179)$$

$$= \frac{I_0}{2}\,(101 - j527)$$

$$= .0001\,(101 - j527)$$

$$= .0101 - j.0527$$

$$E_0' = \frac{I_0}{2}\,(567 - j348) + (668 - j179)$$

$$= \frac{I_0}{2}\,(1235 - j169)$$

$$= .0001\,(1235 - j169)$$

$$= .1235 - j.0169$$

A comparison of the values of I_0'' and E_0'' at 800 cycles under the conditions that the opeh core transformer is replaced by one with a closed core and when the telephone set as now connected is used shows the following results,

$I_0'' = .000034 + j.0000697$ with closed core transformer

$I_0'' = .0000483 - j.0000227$ with open core transformer

$E_0'' = .0101 - j.0527$ with closed core transformer

$E_0'' = .0368 + j.0062$ with open core transformer

There is a material decrease in the values of I_0'' and E_0'' and hence this method does produce better results than the open core transformer. Since nothing but a transformer and a capacity were placed in the circuit, the power relations must remain the same as before. Since the replacing of the coil by a transformer with a closed core added when proportioned to give zero reflected wave at 1200 cycles, it remains only to select the transformer to be used in such a manner that the best results at all frequencies will be obtained.

As a result of many calculations to determine at which frequency the replacing of the open core transformer with one of closed core would do the most good at all frequency, it was found that the transformer should be of such a ratio that at 800 cycles the value of $r_1 + jx_1$ would equal $h + jK$. Under the conditions specified the transformer must have a ratio of

$$\sqrt{\frac{668 - 30}{156}} = \sqrt{4.09} = 2.02$$

and the capacity to be inserted in the secondary circuit must be

$$\frac{10^6}{2\pi fC} = 154 - \frac{179}{4.08} = 110.2$$

$$C = 1.81$$

When the open core transformer is thus replaced by one with a closed core and a capacity of 1.81 micro-farads is placed in the secondary circuit, the reflected waves are entirely eliminated at 800 cycles and at 1200 cycles they are reduced 35 %. The

power available to actuate the receiver remains the same for 800 cycles and is increased to 135 % of the value with the open core transformer at 1200 cycles. These conditions at 1600 cycles are found to be improved 25 %. Thus this method is an improvement over the method commonly used.

It has been shown in the preceding paragraphs that there are three methods by which the reflected waves can be eliminated at one frequency and reduced at others. These methods are;- First to insert a capacity and resistance in series with the telephone set, Second, to insert a capacity in series with the telephone set and use a transformer to make $r_1 + jx_1$ equal to $h + jK$. Third to replace the open core transformer, or induction coil as it is called, by a closed core transformer. The first of these methods was found to be impracticable due to the added loss of power in the resistance. The second method was found impracticable becauseit would be used only on a local battery telephone. The third method, however, was found to produce a great improvement because it eliminated the reflected waves at one frequency and reduced them at other frequencies. Thus an improvement has been effected by simply replacing the open core transformer by one with a closed core having a ratio of one to two and two hundreths (1 to 2.02), and replacing the 2 micro-farad condenser by one of 1.81 micro-farads capacity.

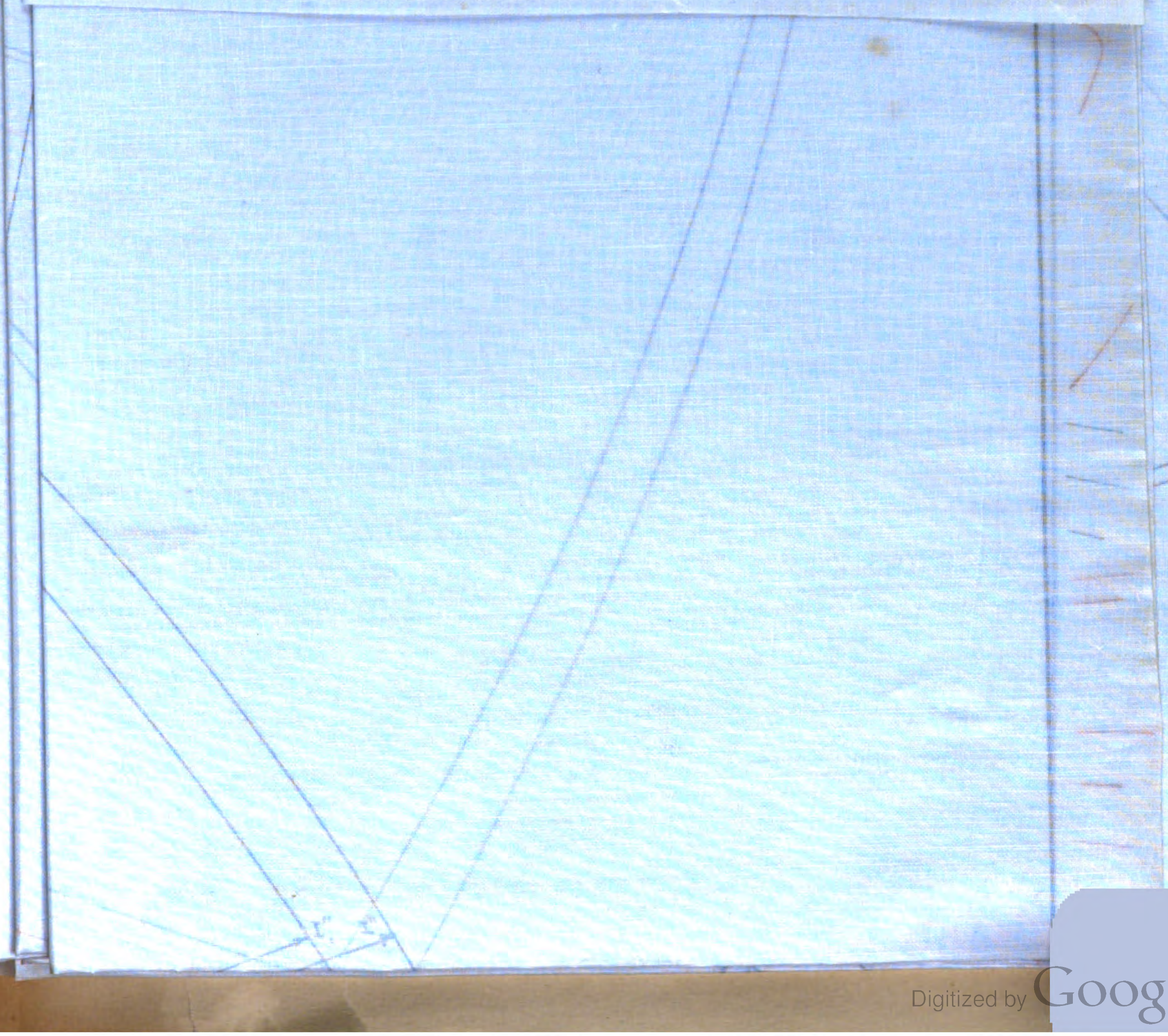
In the preceding chapters of this thesis an attempt has been made to present in logical order the steps necessary in a study of the effects of terminal constants on telephone transmission. In the first chapter a short history of the improvements made up to this time and the possibilities still open were given. in Chapter II, the equations for the current and voltage at any point in the line were derived in detail and discussed at some length. The next chapter, Chapter III, gave in detail the methods used for securing frequencies of the same order as the most important speach frequencies, In this chapter it was also shown how the measurements to determine the resistance and inductance of the various elements of the telephone set were made at the high frequencies required. Chapter IV contains the results, in tabular form, of the measurements made and of the calculations made to show the current and voltage relations along the line. Chapter IV also contains the curves drawn from these tables.

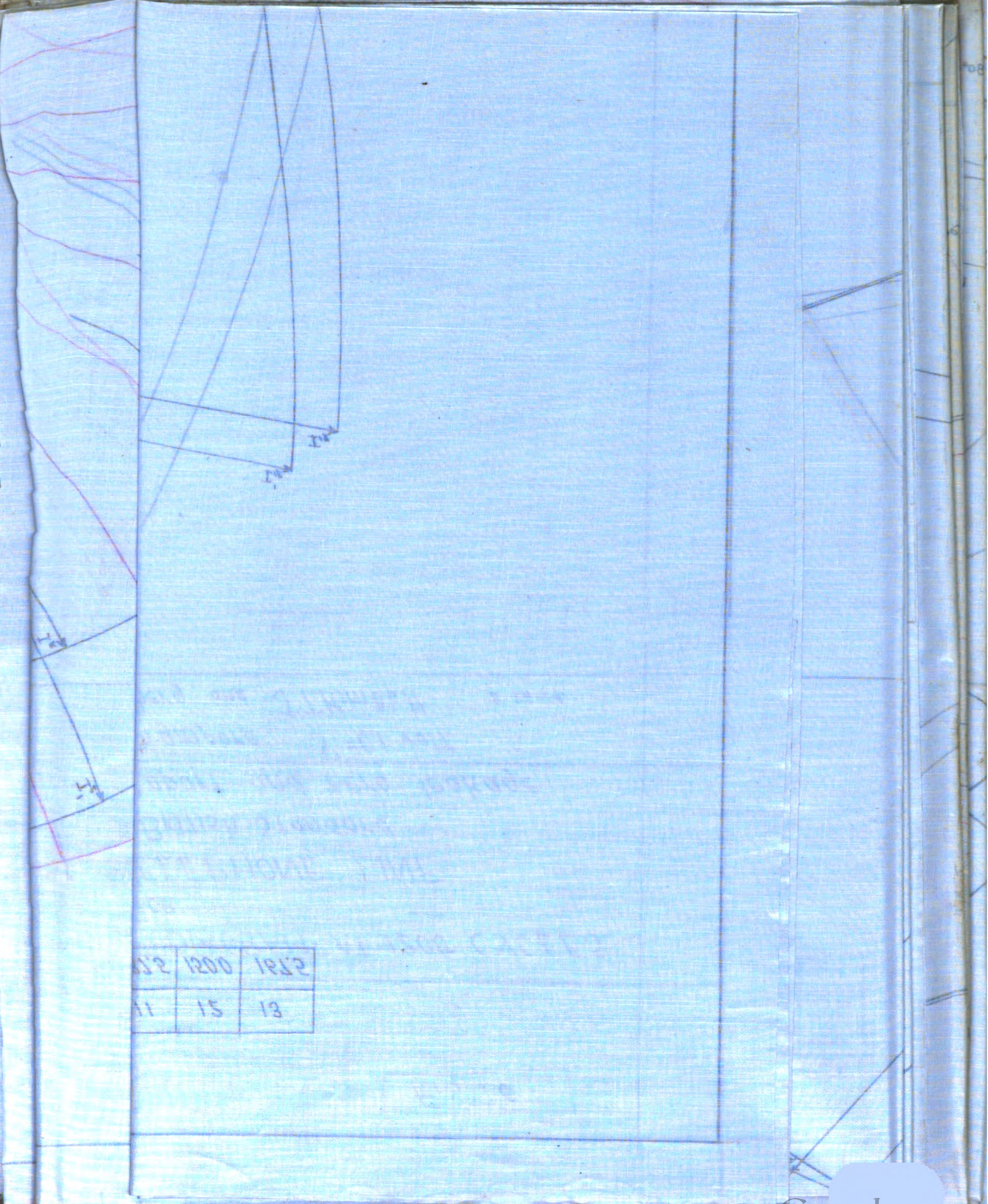
In this, the last chapter of this thesis, calculations are shown which show that the greatest possibility for an improvement lies in replacing the open core transformer now in use by one with a closed core.

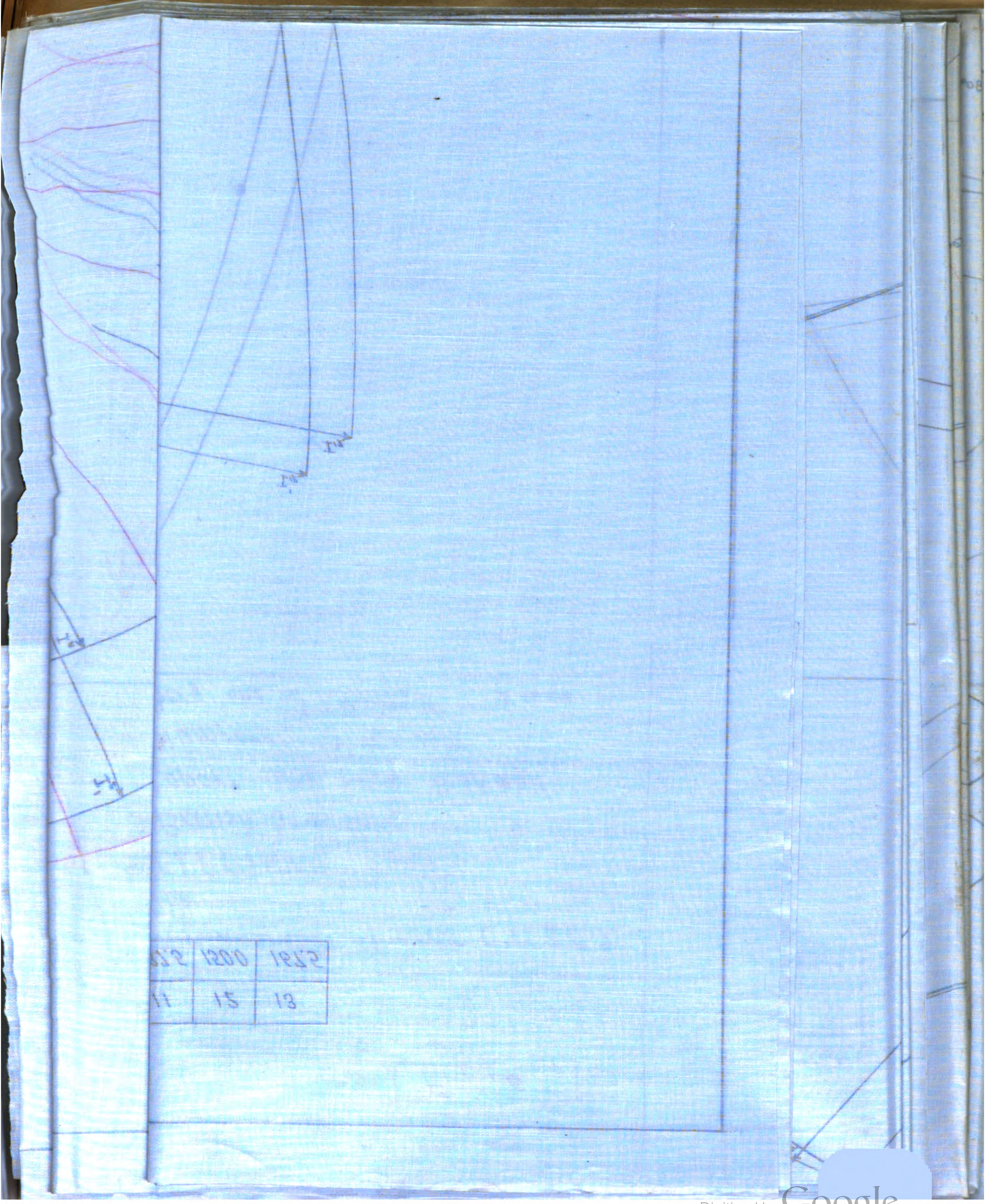
Approved

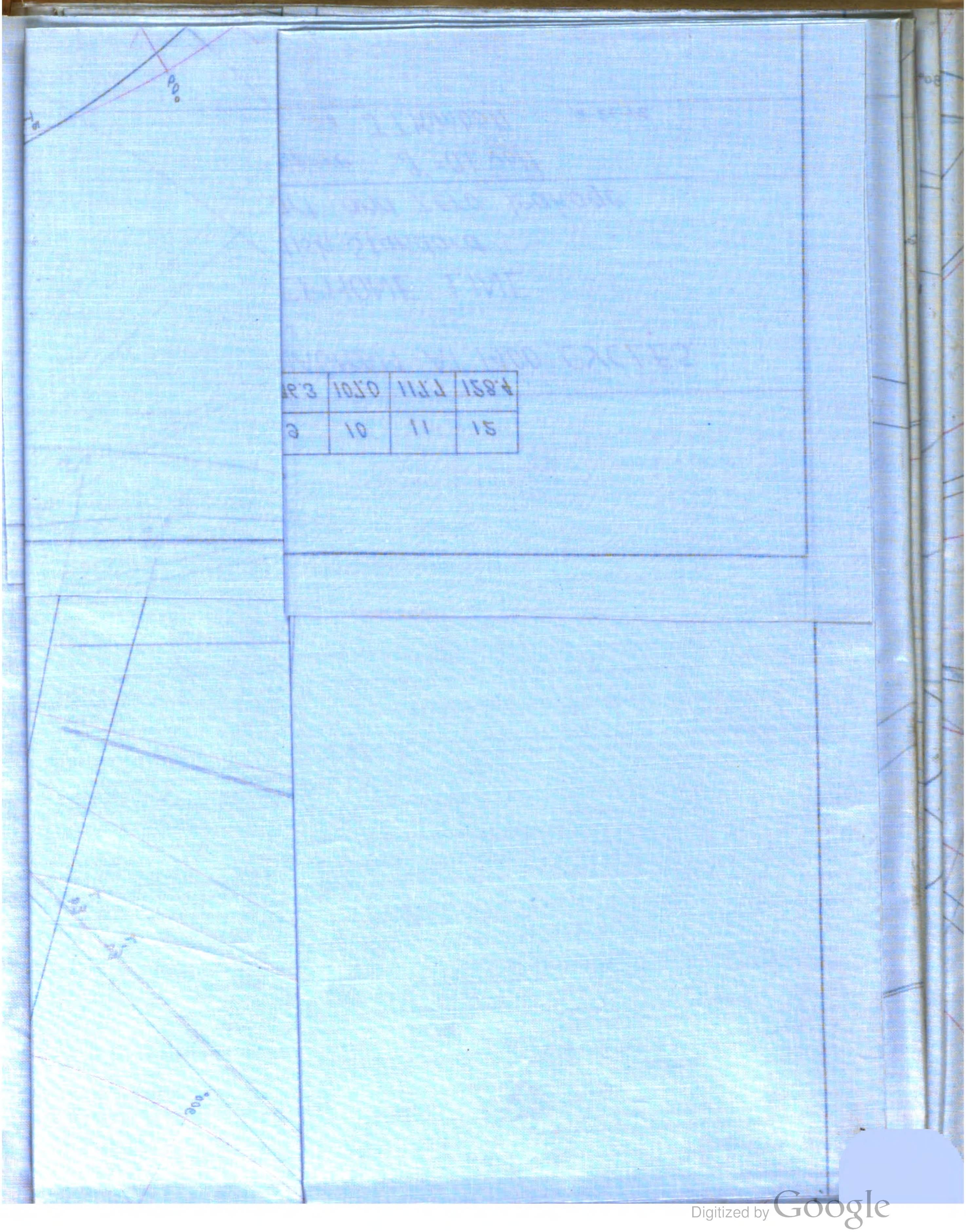
Geo H Gray

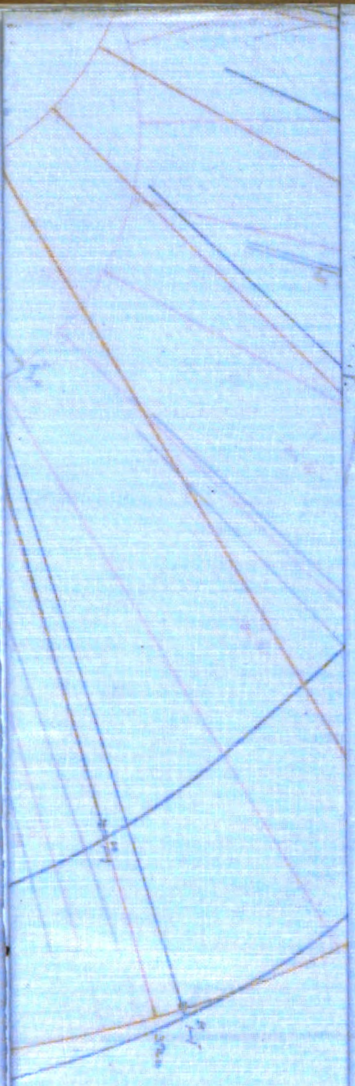
Edward Bennett

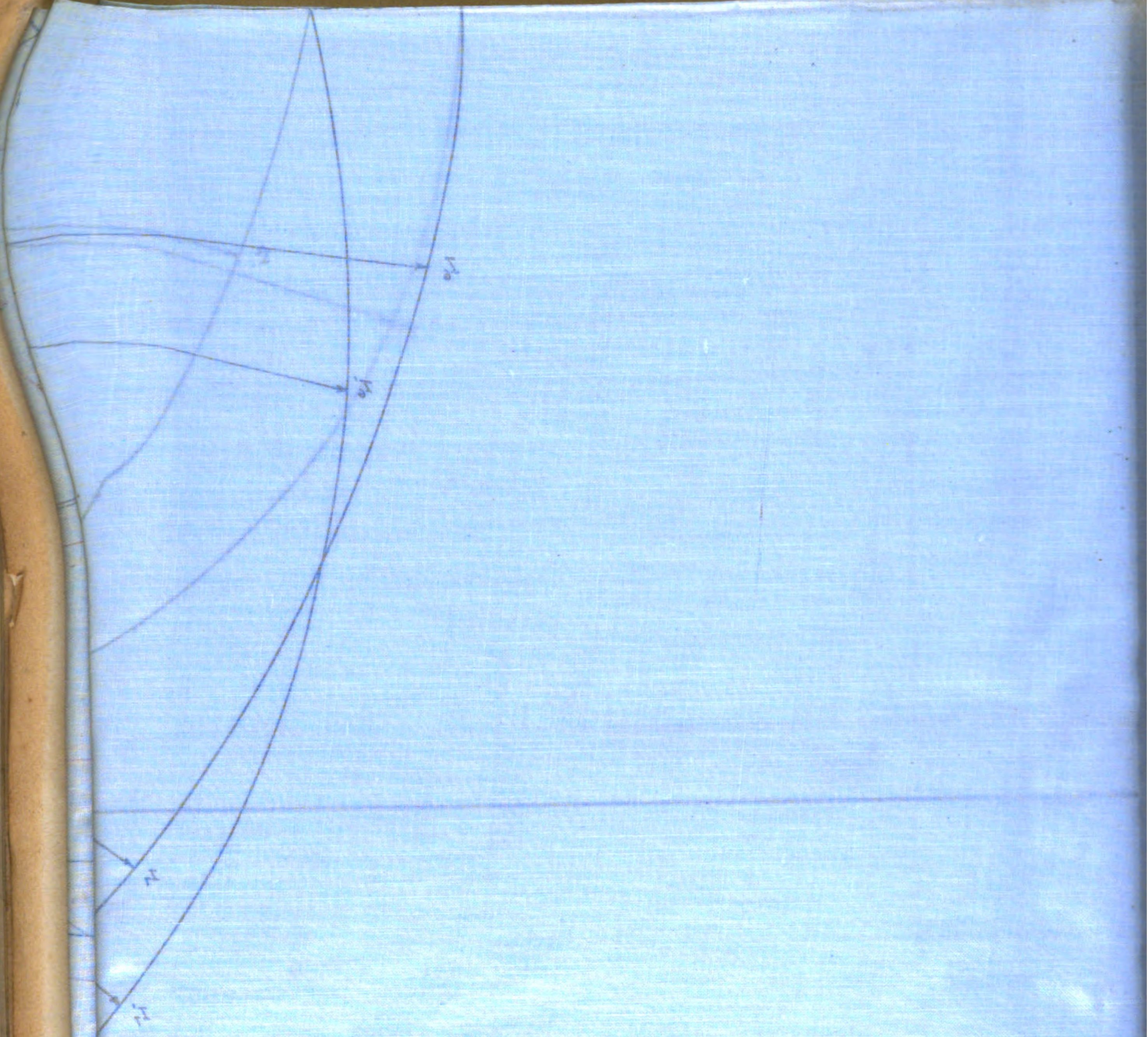


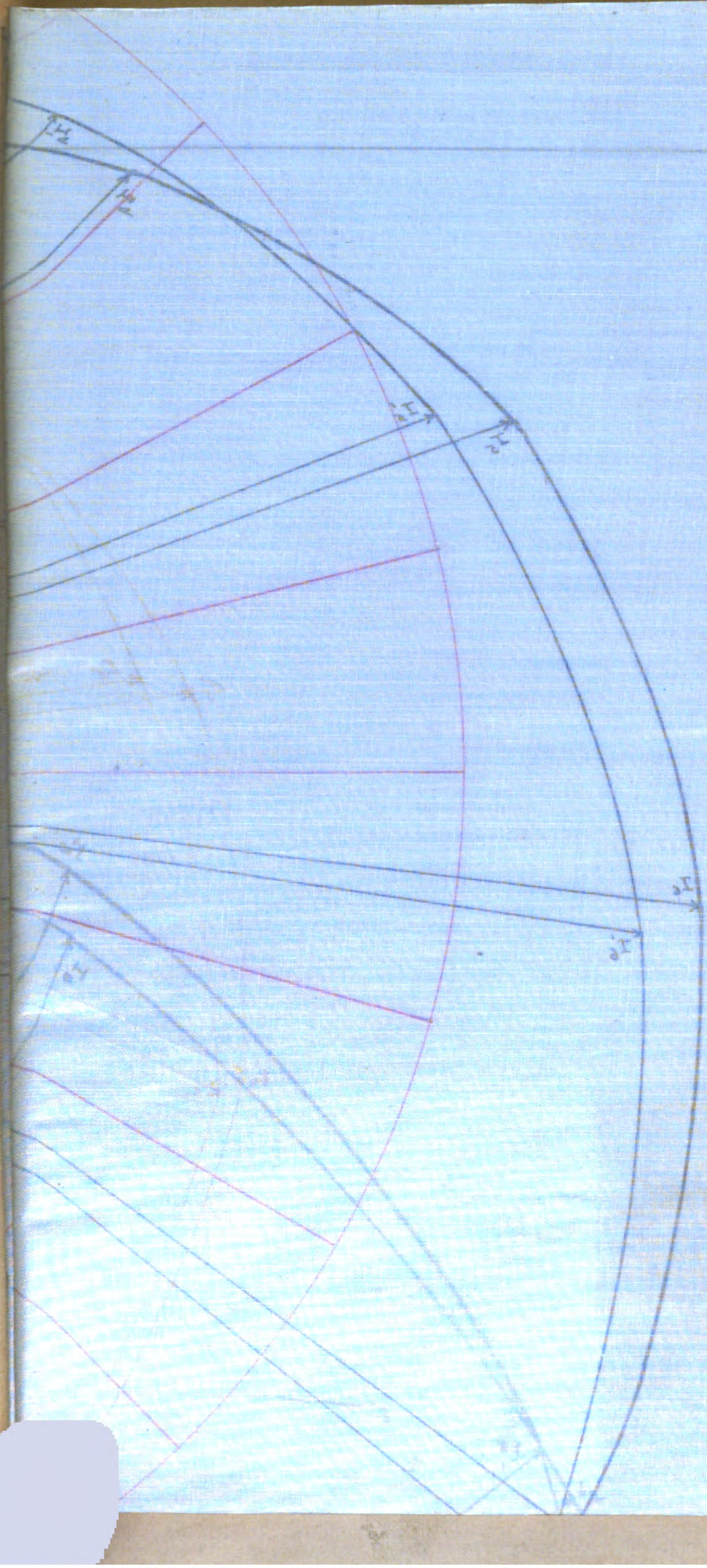












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